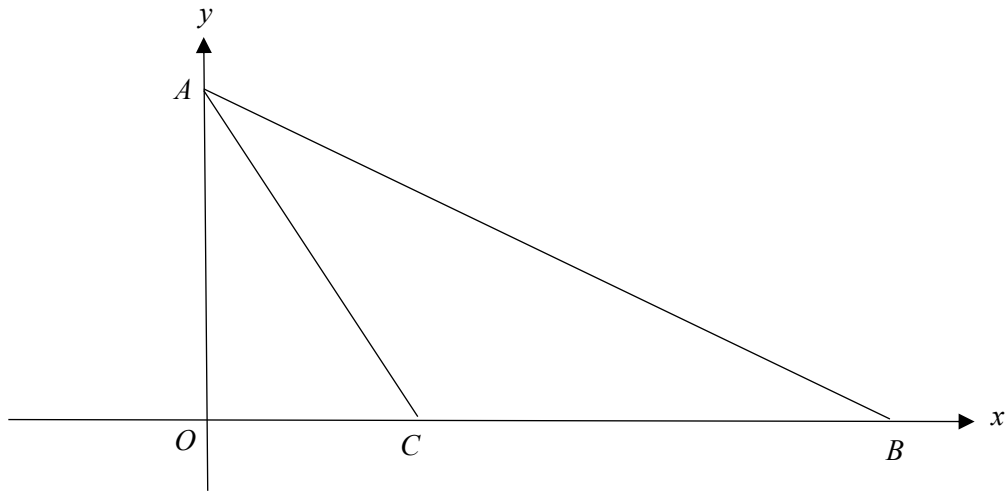


Elementary Math Topical (**Coordinate Geometry**)

Question 1:

The diagram shows a sketch of a straight line AB with equation $5y + 2x = 35$. The length of BC is 10 units.



- Show that the coordinates of A and B are (0,7) and (17.5,0) respectively.
- Find the length of AB .
- Find the equation of the line AC .
- Find the shortest distance from C to the line AB .
- The line $y = 2x - 1$ intersects AB at D . Find the coordinates of D .

a)

$$5y + 2x = 35$$

$$\text{when } x = 0,$$

$$5y = 35$$

$$y = 7$$

$$\therefore A(0, 7)$$

$$\text{when } y = 0$$

$$2x = 35$$

$$x = 17.5$$

$$\therefore B(17.5, 0)$$

EQUITY

LEARNING PLACE

Elementary Math Topical (**Coordinate Geometry**)

b)

$$\text{Length } AB = \sqrt{17.5^2 + 7^2} = 18.848 \text{ units} = 18.8 \text{ units}$$

c)

$$C(7.5, 0)$$

$$\text{Grad } AC = \frac{7 - 0}{0 - 7.5} = -\frac{14}{15}$$

Eqn of AC:

$$y = -\frac{14}{15}x + 7$$

d)

$$\frac{1}{2}(10)(7) = \frac{1}{2}(x)(18.848)$$

$$x = 3.71 \text{ units}$$

e)

Sub $y = 2x - 1$ into equation of AB

$$5(2x - 1) + 2x = 35$$

$$10x - 5 + 2x = 35$$

$$12x = 40$$

$$x = \frac{10}{3}$$

$$y = \frac{17}{3}$$

$$D\left(\frac{10}{3}, \frac{17}{3}\right)$$

Elementary Math Topical (**Coordinate Geometry**)

Question 2:

The point P is $(0, k - 3)$ and it lies on the line, l , with gradient = $-\frac{2}{5}$ and y-intercept = 2.

a) Find the value of k .

b) Given that $Q(-2, n)$ lies on the line l , find the value of n .

a)

$$\text{Eqn of } l: y = -\frac{2}{5}x + 2$$

$$k - 3 = 2$$

$$k = 5$$

b) sub $x = -2$ and $y = n$

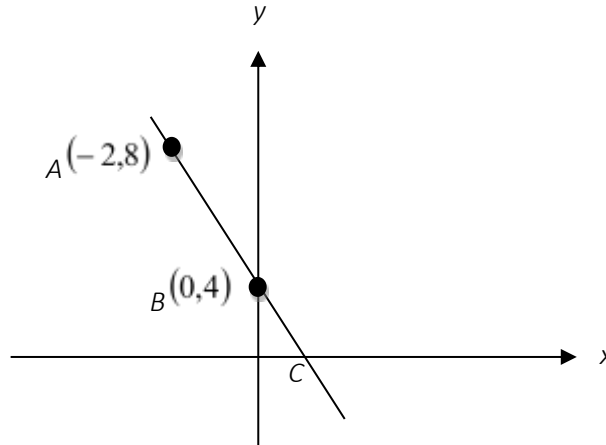
$$n = -\frac{2}{5}(-2) + 2$$

$$n = \frac{14}{5}$$

Elementary Math Topical (**Coordinate Geometry**)

Question 3:

The diagram shows a straight line passing through the points $A(-2,8)$, $B(0,4)$ and C .



a) Calculate the length of the line joining A to B .

b) Find the equation of the line.

c) Write down the coordinates of point C .

a)

$$\text{Length } AB = \sqrt{(-2 - 0)^2 + (8 - 4)^2} = 4.47 \text{ units}$$

b)

$$\text{Grad } AB = \frac{8 - 4}{-2 - 0} = -2$$

Eqn of AB

$$y = -2x + 4$$

c)

when $y = 0$

$$0 = -2x + 4$$

$$2x = 4$$

$$x = 2$$

$$C(2, 0)$$

Elementary Math Topical (**Coordinate Geometry**)

Question 4:

The coordinates of points A and B are $(-2, 5)$ and $(2.5, -4)$ respectively.

- a) Calculate the distance between the two points.
b) If a line passes through the two points, find the equation of the line.

a)

$$\text{Length } AB = \sqrt{(-2 - 2.5)^2 + (5 - (-4))^2} = 10.1 \text{ units}$$

b)

$$\text{Grad } AB = \frac{5 - (-4)}{-2 - 2.5} = -2$$

Eqn of AB

$$-4 = (2.5)(-2) + c$$

$$c = 1$$

$$y = -2x + 1$$

Elementary Math Topical (**Coordinate Geometry**)

Question 5:

Line p passes through the points $A(-1, -6)$ and $B(3, 4)$.

- Find the length of AB .
- Find the gradient of the line p .
- Find the equation of line p .
- Verify if the point $(-1, 27)$ lies on the line p .

a)

$$\text{Length } AB = \sqrt{(3 - (-1))^2 + (4 - (-6))^2} = 10.8 \text{ units}$$

b)

$$\text{Grad } AB = \frac{4 - (-6)}{3 - (-1)} = \frac{5}{2}$$

c)

Eqn of p :

$$4 = \left(\frac{5}{2}\right)(3) + c$$

$$c = -\frac{7}{2}$$

$$y = \frac{5}{2}x - \frac{7}{2}$$

d) Let $x = -1$

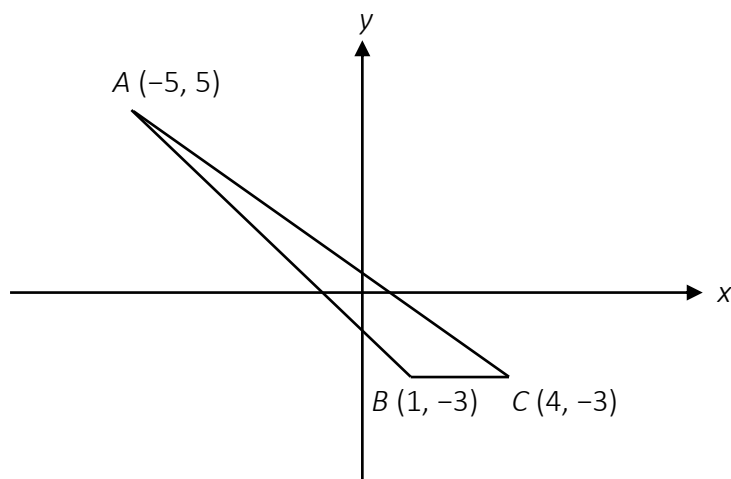
$$y = -\frac{5}{2} - \frac{7}{2} = -6$$

The point does not lie on the line since the line passes through $(-1, -6)$.

Elementary Math Topical (**Coordinate Geometry**)

Question 6:

In the diagram, points $A(-5, 5)$, $B(1, -3)$ and $C(4, -3)$ form an obtuse triangle.



Find

- the length AB ,
- the coordinates of D such that $ABCD$ is a parallelogram
- the equation of line AB ,
- the coordinates of M and of N given that AB cuts the x -axis and y -axis at M and N respectively,
- the area of triangle ABC ,
- the exact value of $\cos ABC$.

a)

$$\text{Length } AB = \sqrt{(5 - (-3))^2 + (-5 - 1)^2} = 10 \text{ units}$$

b) By observation BC is horizontal with length 3 so AD is also horizontal with length 3

$$D(-2, 5)$$

c)

$$\text{Gradient of } AB = \frac{5 - (-3)}{-5 - 1} = -\frac{4}{3}$$

EQUITY

LEARNING PLACE

Elementary Math Topical (Coordinate Geometry)

Eqn AB

$$-3 = -\frac{4}{3}(1) + c$$

$$c = -\frac{5}{3}$$

$$y = -\frac{4}{3}x - \frac{5}{3}$$

d) when $x = 0$

$$y = -\frac{5}{3}$$

$$N\left(0, -\frac{5}{3}\right)$$

When $y = 0$

$$\frac{4}{3}x = \frac{5}{3}$$

$$x = \frac{5}{4}$$

$$M\left(\frac{5}{4}, 0\right)$$

e)

$$\text{Area } ABC = \frac{1}{2}(3)(8) = 12 \text{ units}^2$$

f)

$$\cos ABC = -\frac{3}{5}$$

Elementary Math Topical (**Coordinate Geometry**)

Question 7:

The line $6x - 3y = 24$ cuts the x -axis at M and y -axis at N .

a) Find the coordinates of M and N .

b) Another line l has the same gradient as the given line and passes through the point $(1, 5)$. Find the equation of the line l .

a) when $x = 0$

$$-3y = 24$$

$$y = -8$$

$$N(0, -8)$$

When $y = 0$,

$$6x = 24$$

$$x = 4$$

$$M(4, 0)$$

b)

$$-3y = -6x + 24$$

$$y = 2x - 8$$

$$\text{Gradient} = 2$$

Eqn of l :

$$5 = (2)(1) + c$$

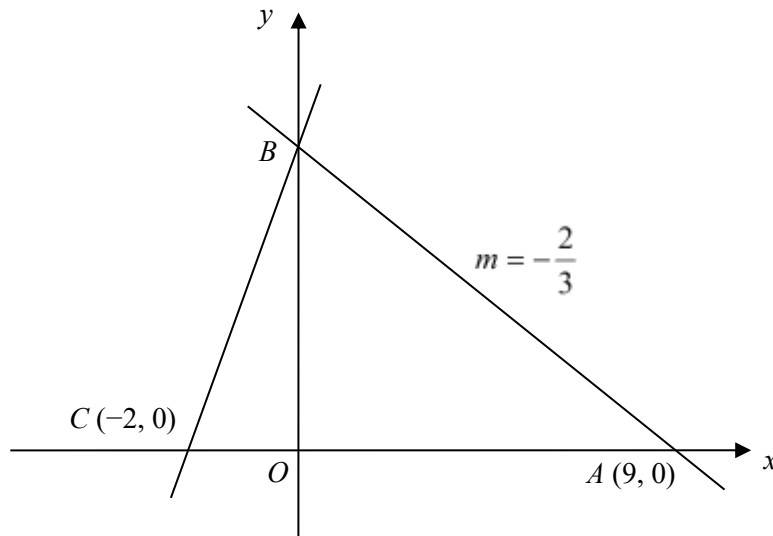
$$c = 3$$

$$y = 2x + 3$$

Elementary Math Topical (**Coordinate Geometry**)

Question 8:

In the diagram, which is not drawn to scale, the coordinates of A and C are (9, 0) and (-2, 0) respectively.



The gradient, m , of the line AB is $-\frac{2}{3}$.

- Find the equation of the line AB .
- Find the equation of the line BC .
- Determine whether triangle ABC is a right-angled triangle.
- Find the area of triangle ABC .

a)

Eqn of AB

$$0 = -\frac{2}{3}(9) + c$$

$$c = 6$$

$$y = -\frac{2}{3}x + 6$$

Elementary Math Topical (**Coordinate Geometry**)

b)

$$\text{Grad } BC = \frac{6 - 0}{0 - (-2)} = 3$$

Eqn BC

$$y = 3x + 6$$

c)

$$\tan CBO = \frac{2}{6}$$

$$\angle CBO = 18.434$$

$$\tan OBA = \frac{3}{2}$$

$$\angle OBA = 26.56$$

Since $\angle OBA + \angle CBO \neq 90^\circ$, triangle ABC is not a right-angled triangle.

d)

$$\text{Area } ABC = \frac{1}{2}(11)(6) = 33 \text{ units}^2$$

Question 9:

A straight line passes through the points $A(0, 16)$ and $B(8, 2)$.

a) Find the equation of line AB .

b) Find the value of n given that $(6, n)$ lies on the line AB .

a)

$$\text{Grad } AB = \frac{16 - 2}{0 - 8} = -\frac{7}{4}$$

$$\text{eqn } AB: y = -\frac{7}{4}x + 16$$

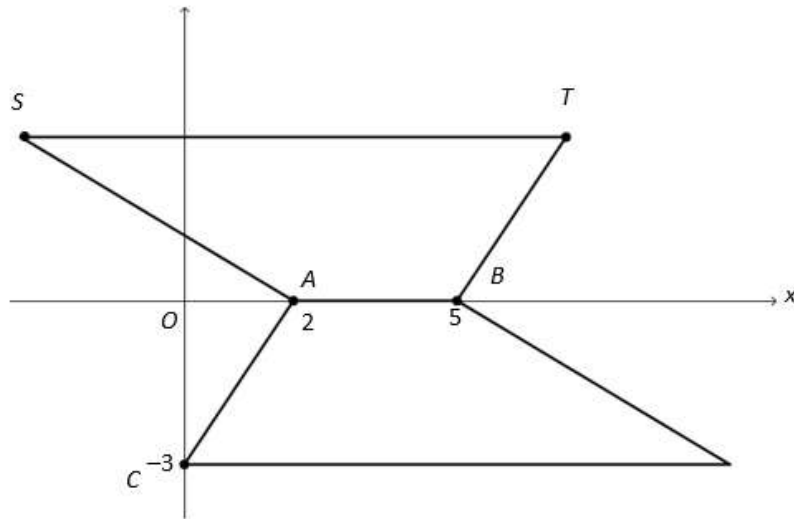
b) Let $x = 6, y = n$

$$n = -\frac{7}{4}(6) + 16$$

$$n = 5.5$$

Elementary Math Topical (**Coordinate Geometry**)

Question 10:



The diagram shows two congruent trapeziums on an xy -plane. The points A and B lie on the x -axis and the point C lies on the y -axis.

- a) Find the coordinates of the point T .
- b) Given that the combined area of both trapeziums is 39 units^2 , find the coordinates of the point S .

a)

$$T(7, 3)$$

b)

$$\frac{1}{2}(3 + ST)(3) = \frac{1}{2}(39)$$

$$3 + ST = 13$$

$$ST = 10$$

$$S(-3, 3)$$

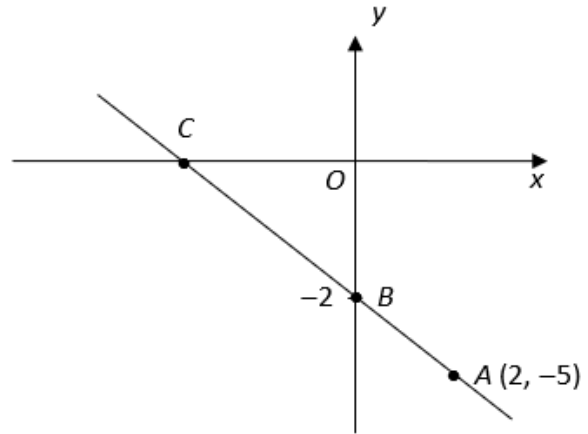
Elementary Math Topical (**Coordinate Geometry**)

Question 11:

The diagram shows a sketch of a straight line passing through the points $A(2, -5)$, B on the y -axis and C on the x -axis.

Find the

- a) equation of the line,
- b) coordinates of C
- c) equation of the line parallel to BC and passing through the origin



a)

$$\text{Grad of } AB = \frac{-5 - (-2)}{2 - 0} = -\frac{3}{2}$$

$$\text{eqn of line: } y = -\frac{3}{2}x - 2$$

b)

$$\text{when } y = 0$$

$$\frac{3}{2}x = -2$$

$$x = -\frac{4}{3}$$

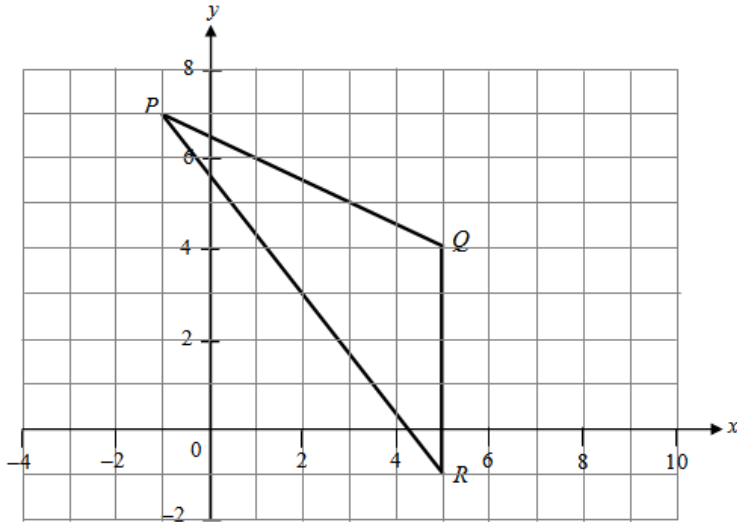
c)

$$y = -\frac{3}{2}x$$

Elementary Math Topical (**Coordinate Geometry**)

Question 12:

The points $P(-1, 7)$, $Q(5, 4)$ and $R(5, -1)$ are shown in the diagram



Find

- a) the gradient of line PQ ,
- b) the equation of the line PQ ,
- c) the length of line PQ ,
- d) $\cos \angle PQR$,
- e) the area of triangle PQR .

a)

$$\text{Grad of } PQ = \frac{7 - 4}{-1 - 5} = -\frac{1}{2}$$

b)

Eqn of PQ :

$$7 = \left(-\frac{1}{2}\right)(-1) + c$$

$$c = \frac{13}{2}$$

$$y = -\frac{1}{2}x + \frac{13}{2}$$

Elementary Math Topical (**Coordinate Geometry**)

c)

$$\text{Length } PQ = \sqrt{(5 - (-1))^2 + (4 - 7)^2} = \sqrt{45} = 6.71 \text{ units}$$

d)

$$\cos PQR = -\frac{3}{\sqrt{45}}$$

e)

$$\text{Area of } PQR = \frac{1}{2}(5)(6) = 15 \text{ units}^2$$

Question 13:

The coordinates of O , A and B are $(0, 0)$, $(5, 0)$ and $(7, 8)$ respectively. Find

a) the gradient of AB ,

b) the equation of AB

c) the area of triangle OAB

d) The distance between the points $(2 - k, 1)$ and $(2, k)$ is $\sqrt{5 - 4k}$ units.
Find the possible values of k .

a)

$$\text{Grad } AB = \frac{8 - 0}{7 - 5} = 4$$

b)

eqn of AB

$$0 = 5(4) + c$$

$$c = -20$$

$$y = 4x - 20$$

c)

$$\text{Area of triangle} = \frac{1}{2}(5)(8) = 20 \text{ units}^2$$

Elementary Math Topical (**Coordinate Geometry**)

d)

$$\sqrt{((2-k-2)^2 + (1-k)^2)} = \sqrt{5-4k}$$

$$k^2 + 1 - 2k + k^2 = 5 - 4k$$

$$2k^2 + 2k - 4 = 0$$

$$k^2 + k - 2 = 0$$

$$(k+2)(k-1) = 0$$

$$k = -2 \text{ or } k = 1$$

Question 14:

The line $3y - 6x + 9 = 0$ cuts the y -axis at P and the x -axis at Q .

a) State the gradient of the line PQ .

b) Find the coordinates of P and Q .

c) Find the equation of another line, l , which is parallel to the line PQ and passes through the point $(1, -8)$.

a)

$$3y = 6x - 9$$

$$y = 2x - 3$$

$$\text{Gradient of } PQ = 2$$

b)

$$\text{when } x = 0$$

$$y = -3$$

$$P(0, -3)$$

$$\text{when } y = 0$$

$$x = \frac{3}{2}$$

$$Q\left(\frac{3}{2}, 0\right)$$

Elementary Math Topical (**Coordinate Geometry**)

c)

eqn of line

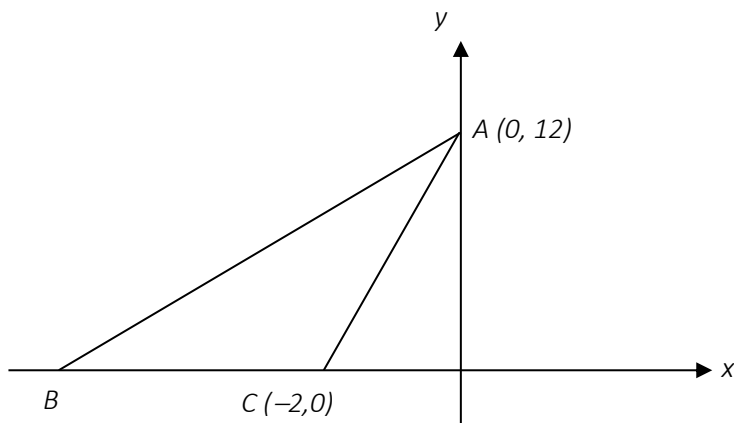
$$-8 = 1(2) + c$$

$$c = -10$$

$$y = 2x - 10$$

Question 15:

In the figure below, B lies on the x -axis. The area of $\triangle ABC$ is 48 units² and the equation of the line AB is $y = ax + b$.



- Find the coordinates of the point B ,
- Find the value of a and of b .
- Show that the points B , D and A are collinear, where D is $(-5, 6)$.

a)

$$\frac{1}{2}(BC)(12) = 48$$

$$BC = 8$$

$$B(-10, 0)$$

Elementary Math Topical (**Coordinate Geometry**)

b)

$$\text{Grad } AB = \frac{12 - 0}{0 - (-10)} = \frac{6}{5}$$

$$\therefore a = \frac{6}{5}, \quad b = 12$$

c)

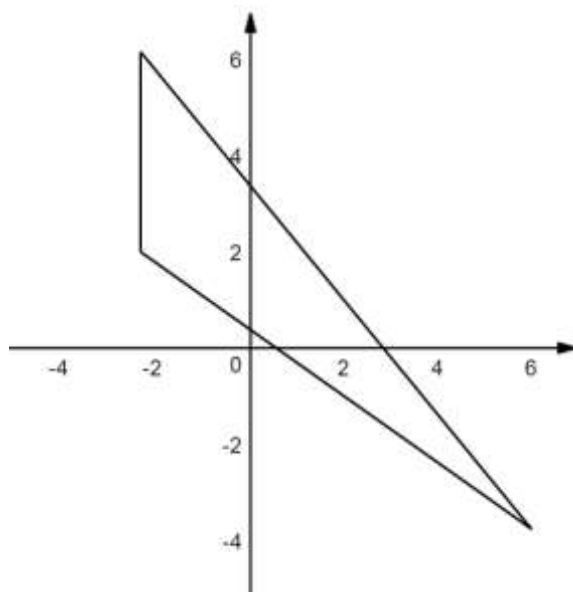
$$\text{Grad } AD = \frac{6 - 12}{-5 - 0} = \frac{6}{5}$$

Since the gradient is the same and A is a common point, B, D, A are collinear.

Elementary Math Topical (**Coordinate Geometry**)

Question 16:

The diagram below shows three labelled points $A(-2,6)$, $B(-2,2)$ and $C(6,-4)$. D is a point where the line AC meets the x -axis.



Find

- the gradient of the line AC and hence the equation of AC
- the coordinates of D and **hence** the acute angle which the line AD makes with the x -axis,
- the **exact** value of $\cos \angle ABC$
- the area of triangle ABC

a)

$$\text{Grad } AC = \frac{6 - (-4)}{-2 - 6} = -\frac{5}{4}$$

Eqn of AC

$$6 = -\frac{5}{4}(-2) + c$$

$$c = \frac{7}{2}$$

$$y = -\frac{5}{4}x + \frac{7}{2}$$

EQUITY

LEARNING PLACE

Elementary Math Topical (Coordinate Geometry)

b) when $y = 0$

$$\frac{5}{4}x = \frac{7}{2}$$

$$x = \frac{14}{5}$$

$$D\left(\frac{14}{5}, 0\right)$$

$$\tan \theta = \frac{5}{4}$$

$$\theta = 51.3^\circ$$

c)

$$\text{Length } BC = \sqrt{(-4 - 2)^2 + (6 - (-2))^2} = 10$$

$$\cos ABC = -\frac{3}{5}$$

d)

$$\text{Area of triangle } ABC = \frac{1}{2}(4)(8) = 16 \text{ units}^2$$

Elementary Math Topical (**Coordinate Geometry**)

Question 17:

The line $4x + 2y = 14$ crosses the x-axis at the point A.

- a) Find the gradient of the line.
- b) Find the coordinates of point A.

a)

$$2y = -4x + 14$$

$$y = -2x + 7$$

Gradient of line is 7

b) when $y = 0$

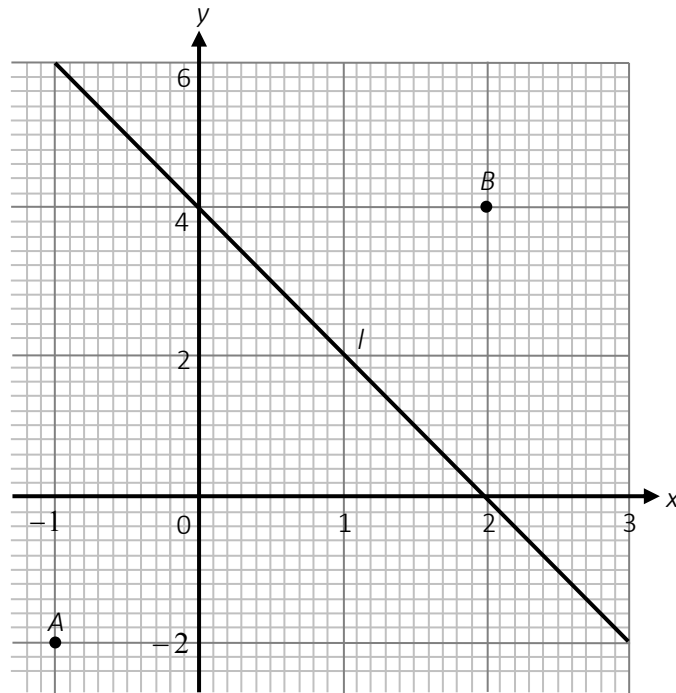
$$4x = 14$$

$$x = \frac{7}{2}$$

$$A\left(\frac{7}{2}, 0\right)$$

Elementary Math Topical (Coordinate Geometry)

Question 18:



a) Calculate the length of AB .

b) Calculate the gradient of the line l .

c) Find the equation of line l .

a)

$$\text{Length } AB = \sqrt{(2 - (-1))^2 + (4 - (-2))^2} = 6.71 \text{ units}$$

b)

$$\text{Gradient} = \frac{4 - (-2)}{2 - (-1)} = 2$$

c)

Eqn of l

$$4 = 2(2) + c$$

$$c = 0$$

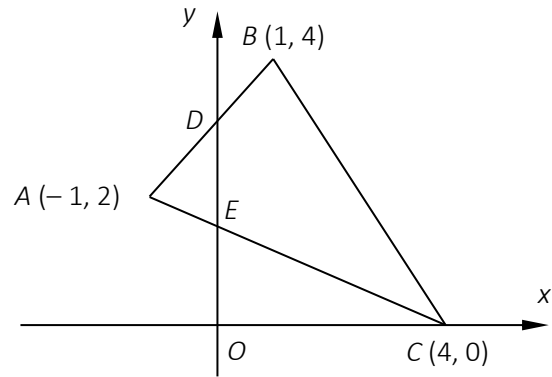
$$y = 2x$$

Elementary Math Topical (**Coordinate Geometry**)

Question 19:

In the diagram, O is the origin, A is $(-1, 2)$, B is $(1, 4)$, C is $(4, 0)$. D and E are points of intersections where AB and AC cut the y -axis.

- a) Find the length of BC .
- b) Determine the equation of line AC .
- c) State the value of $\tan \angle ACO$ as a fraction.
- d) Calculate the value of $\frac{\text{area of triangle } ADE}{\text{area of triangle } ABC}$.



a)

$$BC = \sqrt{(4 - 1)^2 + (0 - 4)^2} = 5 \text{ units}$$

b)

$$\text{Gradient } AC = \frac{2 - 0}{-1 - 4} = -\frac{2}{5}$$

Eqn of AC

$$0 = 4 \left(-\frac{2}{5} \right) + c$$

$$c = \frac{8}{5}$$

$$y = -\frac{2}{5}x + \frac{8}{5}$$

c)

$$\tan \angle ACO = \frac{2}{5}$$

EQUITY

LEARNING PLACE

Elementary Math Topical (Coordinate Geometry)

d)

$$\text{when } x = 0, y = \frac{8}{5}$$

$$D(0,3)$$

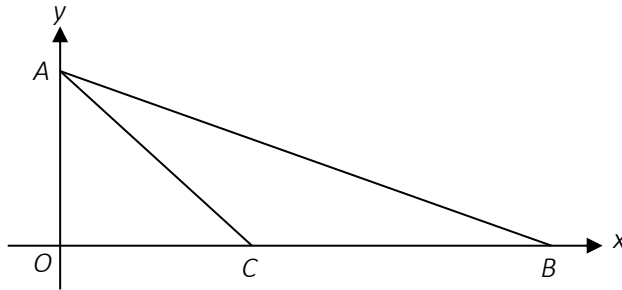
$$\frac{AD}{AB} = \frac{1}{2}$$

$$\frac{AE}{AC} = \frac{1}{4}$$

$$\begin{aligned}\frac{\text{Area of } ADE}{\text{Area of } ABC} &= \frac{\frac{1}{2}(AD)(AE) \sin \angle DAE}{\frac{1}{2}(AB)(AC) \sin \angle DAE} \\ &= \frac{(AD)(AE)}{(AB)(AC)} \\ &= \frac{1}{2} \left(\frac{1}{4} \right) \\ &= \frac{1}{8}\end{aligned}$$

Elementary Math Topical (**Coordinate Geometry**)

Question 20:



In the diagram, the equation of the line AB is $3y + x = 18$ and the length of BC is 12 units. Write down

- a) the coordinates of A
- b) the coordinates of B
- c) Find the length of AB .

Q is a point on the line AB with y -coordinate k . It is given that the gradient of the line CQ is $\frac{4}{5}$.

- d) Find the value of k .
- e) **Hence**, calculate the area of triangle CQB .

a)

$$3y = -x + 18$$

$$y = -\frac{1}{3}x + 6$$

$$A(0, 6)$$

b)

$$\text{when } y = 0$$

$$x = 18$$

$$B(18, 0)$$

c)

$$\text{Length of } AB = \sqrt{(0 - 18)^2 + (6 - 0)^2} = 18.974 = 19.0 \text{ unit}$$

EQUITY

LEARNING PLACE

Elementary Math Topical (Coordinate Geometry)

d)

$$C(6, 0)$$

Eqn of CQ

$$0 = \frac{4}{5}(6) + c$$

$$c = -\frac{24}{5}$$

$$y = \frac{4}{5}x - \frac{24}{5}$$

$$y = -\frac{1}{3}x + 6$$

Solve simultaneously

$$\frac{4}{5}x - \frac{24}{5} = -\frac{1}{3}x + 6$$

$$\frac{17}{15}x = \frac{54}{5}$$

$$x = \frac{162}{17}$$

$$y = \frac{48}{17}$$

$$\therefore k = \frac{48}{17}$$

e)

$$\text{Area of triangle} = \frac{1}{2}(12)\left(\frac{48}{17}\right) = \frac{288}{17} \text{ units}^2$$