EQUITY
LEARNING PLACE

Sec 4 June 2023 Additional Math Revision IV

Trigonometry (I): Basic Angle

Summary:

- Basic Angle Concept
- Quadrants (A, S, T, C)
- Surds
- Special angles
- Some Trigonometry Identities

Example: Given that $2\operatorname{cosec} x = 2\frac{1}{6}$, where x is obtuse, find, without using a calculator, the exact value of

a) $\cot x$,

b) $\cos 2x$

Practice Questions

1) Given that θ is acute and $\sin\theta = s$, express, in terms of s,

a) $\tan\theta$ [2]

b) $\sec\theta$ [1]

2) A and B are acute angles such that $\sin(A-B) = \frac{3}{8}$ and $\sin A\cos B = \frac{5}{8}$. Without calculating angles A and B, find

a) $\cos A\sin B$ [2]

b) $\sin(A+B)$ [1]

c) $\frac{\cot A}{\cot B}$ [2]

Trigonometry (II): Solving Equation

Summary:

- Basic angle
- Quadrant
- Trigonometry Identities

Example: Solve $4\sin 2x + 3\cos 2x + 1 = 0$ for $0° \leq x \leq 180°$. [4]

Example: Solve the equation $4\sin\theta + 3\cos 2\theta - 1 = 0$ for $0° \leq \theta \leq 360°$ [4]

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Practice Questions

3) Solve $2.5 - \cos 2x = 1.8$ for $0 \le x \le \pi$ [2]

4) Solve the equation $2\cos 2x - 3\sec x + 2 = 0$ for $-\pi \le x \le \pi$. [5]

5) Solve the equation $1 - \dfrac{5}{\sin x} = 2\cot^2 x$ for $-2\pi \le x \le 2\pi$. [7]

Trigonometry (III): Proving

Summary:

- Check angles
- Structure
- Trigonometry Identities
- Algebraic Manipulation

Example:

Show that $\frac{\sin^2 x - 2\sin x}{1-\cos 2x} = \frac{1}{2}(1 - 2\text{cosec } x)$.

Example:

Prove the identity $\text{cosec } 2\theta + \cot 2\theta = \cot \theta$.

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Practice Questions

6) Prove that $\frac{3\cos\theta+\cos 2\theta-1}{\cos^2\theta+2\cos\theta}=2-\sec\theta$. [4]

7) Prove the identity [3]

$$\frac{\tan^2 x}{1-\sec x}+1=-\sec x$$

8) Show that $\frac{\sin x}{\cos x-1}+\frac{\cos x-1}{\sin x}=-\frac{2}{\sin x}$ [4]

Trigonometry (IV): Graph

Summary:

- $y = \sin x$
- $y = \cos x$
- $y = \tan x$: Take note of asymptote.
- $y = \sin ax$: Period, find critical point.
- $y = a \sin x$: Amplitude, identify max and min point.
- $y = \sin x + a$: Graph shifted up or down by a units.

Example: On the same axes, sketch the graphs of $y = 4 \sin \theta - 1$ and $y = -3 \cos 2\theta$ for $0° \leq \theta \leq 360°$.

Example:

a) It is given that $f(x) = 2\cos 6x - \frac{1}{2}$ and $g(x) = 1 - \sin 3x$. State the period of $f(x)$ and $g(x)$, in terms of π. [2]

b) Sketch on the same axes, the graphs of $y = f(x)$ and $y = g(x)$ for $0 \leq x \leq \frac{2\pi}{3}$. [4]

Practice Questions

9) The equation of a curve is $y = 3 - 4\sin 2x$.

a) State the minimum and maximum value of y. [2]

b) Sketch the curve $y = 3 - 4\sin 2x$ for $0 \leq x \leq 2\pi$ [3]

10) In a wave pool, the height of a wave y metres, can be modelled by $y = 2.5 - \cos 2x$, where x represents the horizontal distance of the wave in metres. Sketch the graph $y = 2.5 - \cos 2x$, for $0 \leq x \leq \pi$. [3]

11) On the same axes, sketch the graphs of $y = 3\cos 2x$ and $y = 1 - \sin\frac{1}{2}x$ for $0 \leq x \leq \pi$. [4]

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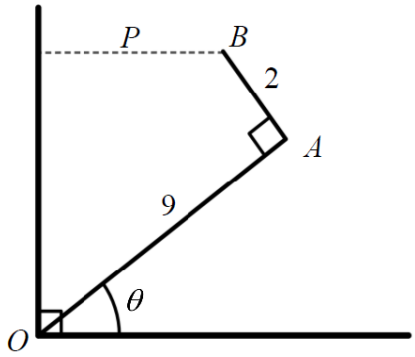
Trigonometry (V): R formula

Summary:

- TOA CAH SOH
- Addition Formula
- Read maximum and minimum from equation.
- $R = \sqrt{a^2 + b^2}$
- $\alpha = \tan^{-1}\left(\frac{b}{a}\right)$

Example:



A L-shaped structure OAB is hinged at the point O. OA is 9 m and AB is 2 m long. OA makes an acute angle, θ, with the ground. Given that P is the perpendicular distance from B to the wall.

a) Show that $P = 9\cos\theta - 2\sin\theta$. [2]

b) Express P in the form $R\cos(\theta + \alpha)$ where $R > 0, 0° < \alpha < 90°$. [4]

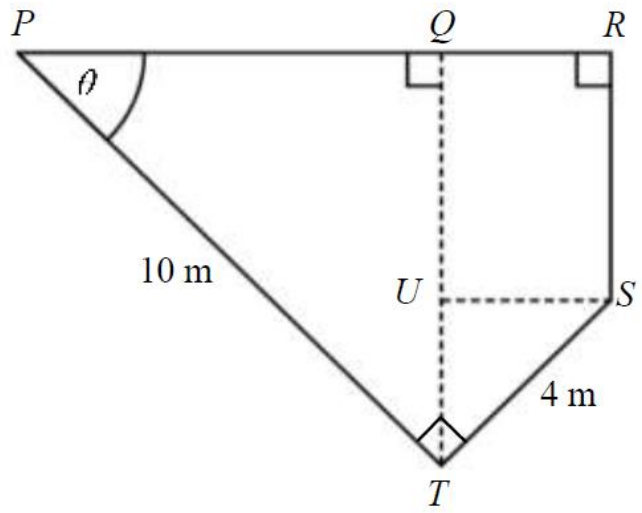
c) State the minimum value of P and find the corresponding value of θ. [3]

d) Find the value of θ when $P = 5$ m. [2]

e) Explain why the maximum value of P is not R. [1]

Practice Questions

12)



The diagram shows a vaccination facility in the shape of a quadrilateral in which angles PQT, PTS and PRS are right angels. SU is parallel to PR. The lengths of PT and ST are 10 m and 4 m respectively. The acute angle QPT is θ radian.

a) Show that the perimeter, W m, is given by $W = 14 + 14\sin\theta + 6\cos\theta$. [2]

b) Find the value of R when $14\sin\theta + 6\cos\theta$ is expressed as $R\sin(\theta + \alpha)$, where R and α are constants and hence state the maximum perimeter of the vaccination facility. [3]

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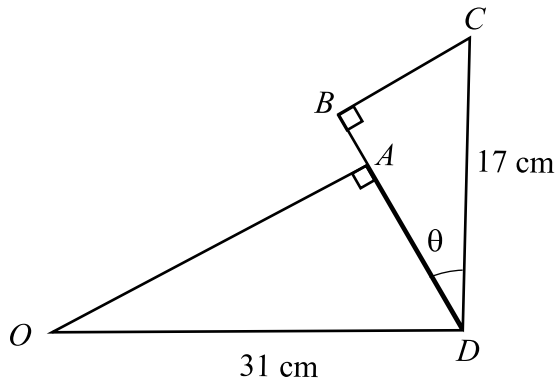
13)



The diagram shows three fixed points *O*, *C* and *D* such that *OD* = 31 cm, *CD* = 17 cm and angle *ODC* = 90°. The lines *OA* and *BC* are perpendicular to the line *BD* which makes an angle θ with the line *CD*. The angle θ can vary in such a way that the point *A* lies between the points *B* and *D*.

a) Show that $OA + AB + BC = 48\cos\theta - 14\sin\theta$. [3]

b) Express $OA + AB + BC$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

c) Find the values of θ for which $OA + AB + BC = 30$. [4]