EQUITY
LEARNING PLACE

Sec 4 June 2023 Additional Math Revision III

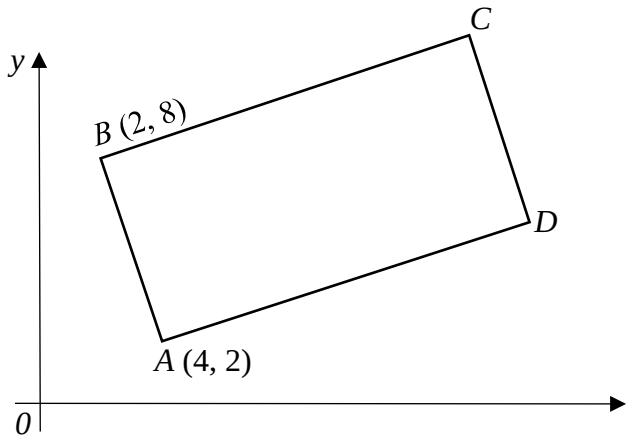
Coordinate Geometry

Summary:

- Distance between 2 points : $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
- Gradient of line : $\frac{y_2 - y_1}{x_2 - x_1}$
- Midpoint : $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- Perpendicular gradient : $m_1 \times m_2 = -1$
- Equation of line : $y = mx + c$
- Area of triangle : $\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$
- Properties of Rhombus : Diagonals are perpendicular and bisect.
- Properties of Parallelogram : Diagonals bisect.

Sec 4 June 2023 Additional Math Revision III

Example:



The diagram shows a rectangle *ABCD* with vertices *A* (4, 2) and *B* (2, 8).

a) Find the equation of *BC*,

b) Given that the line $y = x - 2$ passes through point *C*, find the coordinates of *C*

c) Find the coordinates of the midpoint of *AC*.

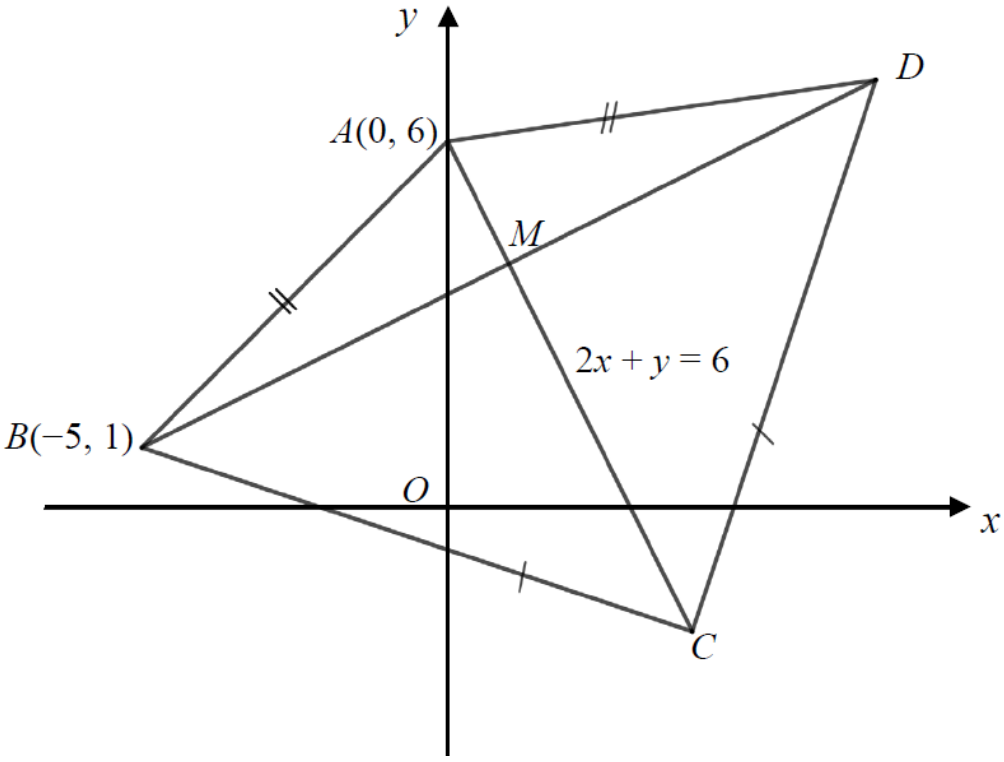
d) Hence, find the coordinates of *D*.

e) Find the area of the rectangle *ABCD*.

Sec 4 June 2023 Additional Math Revision III

Practice Questions

1)



The diagram shows a kite $ABCD$ with $AB = AD$ and $CB = CD$. The diagonals intersect at M. It is given that the coordinates of A and B are $(0\,,6)$ and $(-5\,,1)$ respectively and the equation of AC is $2x + y = 6$. Find

a) the equation of BD. [2]

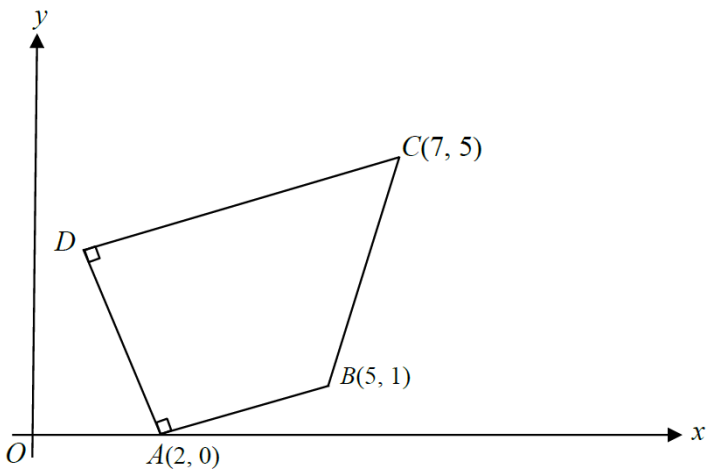
b) the coordinates of M and of D. [4]

Given further that the area of the triangle ABD is $\frac{1}{3}$ of the area of the triangle CBD,

c) find the coordinates of C, [2]

d) find the area of the kite $ABCD$. [2]

2)



The diagram show a quadrilateral $ABCD$ with vertices $A(2,0)$, $B(5,1)$ and $C(7,5)$.

$\angle BAD = \angle ADC = 90°$

a) Find the coordinates of D. [5]

b) A point E lies on CD such that $ABED$ is a square. Find the coordinates of E. [2]

c) Find the area of quadrilateral $ABCD$. [2]

d) A point X on the $x-$ axis is such that $\angle BAX = \angle BXA$. Find the equation of BX. [1]

Circles

Summary:

- Equation of circle : $(x-a)^2+(y-b)^2=r^2$
  $(a\,,b)$ is the centre and r is the radius.
- Perpendicular bisector will pass through centre of circle.
- Tangent is perpendicular to radius.

Example: The points *P* and *Q* both lie on a circle and have the coordinates (2, 0) and $(-2\,,8)$ respectively. The centre of the circle lies on the line $y=x-4$.

a) Find the equation of the perpendicular bisector of *PQ*.

b) Find the equation of the circle.

The point *R* is such that *PR* is a diameter of the circle.

c) Find the coordinates of *R*.

Sec 4 June 2023 Additional Math Revision III

Practice Questions

3) Given that circle *A*, whose equation is $x^2 + y^2 + 4x - 2y - 15 = 0$, has centre *C* and radius *r*.

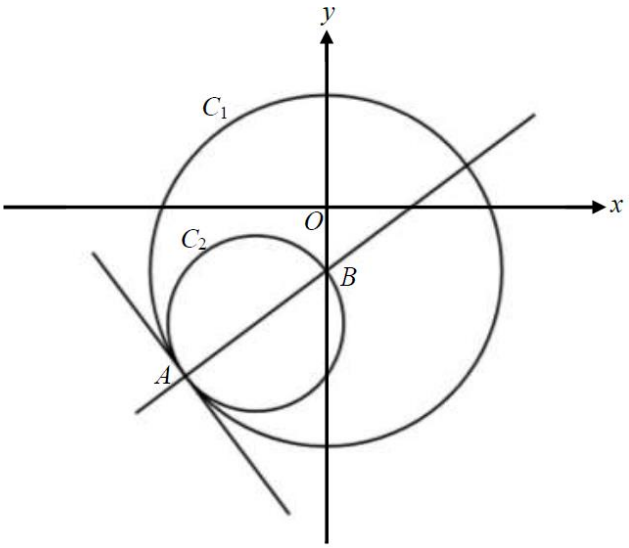
a) Find the coordinates of *C* and the value of *r*. [3]

b) Determine, with reason and working, if $(1\ ,3)$ is a point inside the circle. [2]

c) Find the equation of the tangent to the circle *A* at the point $(2\ ,3)$. [3]

d) The points *M* and *N* lie on the circle. Find the length of the chord *MN* which cuts the *y*-axis. [2]

4)



The diagram shows two circles C_1 and C_2. The equation of circle C_1, with centre B, is $x^2 + y^2 + 8y - 84 = 0$.

The tangent to circle C_1 at the point A has a gradient of $-\frac{4}{3}$.

Circle C_2 has a diameter of AB.

a) Find the radius of circle C_1 and the coordinates of its centre. [2]

b) Find the coordinates of *A*. [4]

c) Find the equation of another circle C_3, which is a reflection of circle C_2 about the $y-$ axis. [3]

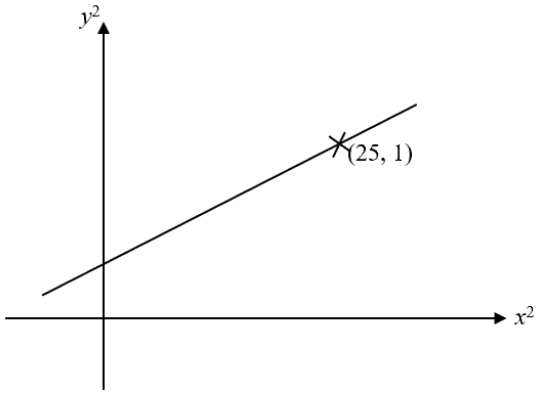
Sec 4 June 2023 Additional Math Revision III

Linear Law

Summary:

- Changing the vertical and horizontal axes.
- Form $Y = mX + c$

Example:



Two variables *x* and *y* are related by the equation $\frac{x^2}{p^2} = 1 + \frac{3y^2}{q^2}$, where *p* and *q* are constants. When the graph of y^2 against x^2 is drawn, a straight line is obtained. Given that the line passes through the point (25, 1) and has a gradient $\frac{1}{15}$, find

a) the exact values of *p* and *q*. [4]

b) the values of *x* when $y = \sqrt{\frac{2}{5}}$. [2]

Sec 4 June 2023 Additional Math Revision III

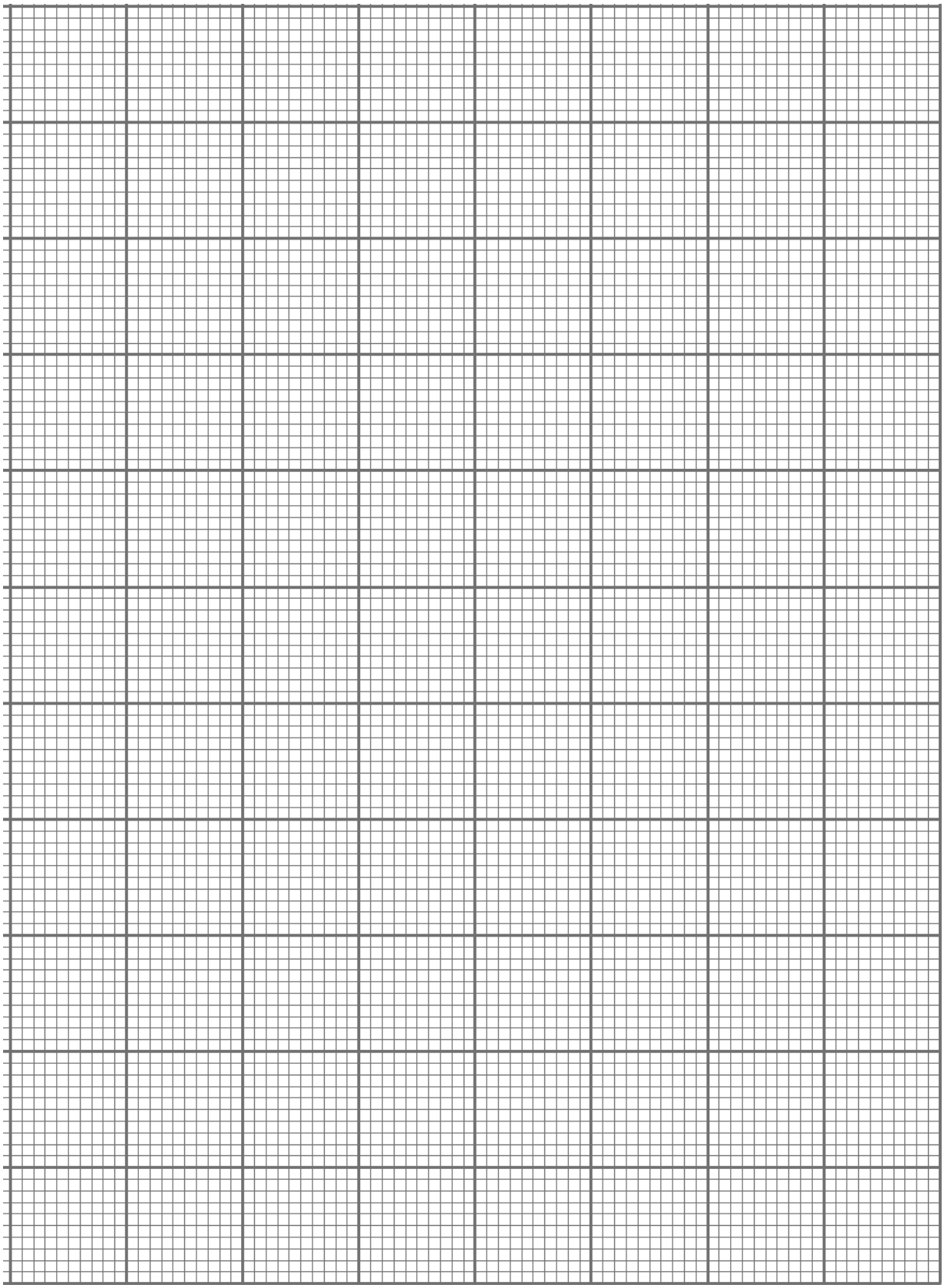
Example: The table below shows experimental values of the variables x and y which are related by the equation $y = pq^x$, where p and q are constants.

x	2	4	6	8	10
y	9.8	19.4	37.4	74.0	144.4

a) Plot $\lg y$ against x for the given data and draw a straight line graph. [2]

b) Use your graph to estimate the value of p and of q. [4]

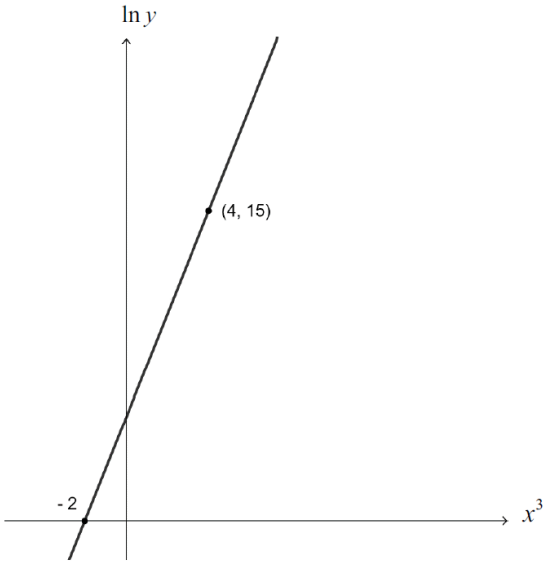
c) On the same diagram, draw the line representing $y = 2^x$ and hence estimate the value of x for which $pq^x = 2^x$. [3]



Sec 4 June 2023 Additional Math Revision III

Practice Question

5) The figure below shows part of a straight line obtained by plotting $\ln y$ against x^3 and it passes through the horizontal axis at -2 and $(4, 15)$.



a) Express y in terms of x. [4]

b) Find the value of x when $y = 11150$. [2]

6) The table below shows the experimental values of two variables *x* and *y*.

x	1.2	1.4	1.6	1.8	2.0
y	4.43	5.88	7.68	10.98	12.00

It is known that *x* and *y* are related by the equation $y = px^q$, where *p* and *q* are constants.

a) Plot lg *y* against lg *x* and draw a straight line graph. [2]

b) There is an incorrect recording of *y* value in the table. Determine the incorrect value and use your graph to estimate the value of *y* to replace the incorrect recording. [2]

c) Use your graph to estimate the value of *p* and *q*. [4]

d) Estimate the value of *x* when *y* is 5. [2]