EQUITY
LEARNING PLACE

Sec 4 June 2023 Additional Math Revision II

Exponential Function

Summary

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(ab)^m = a^m b^m$
- $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
- $a^{-m} = \frac{1}{a^m}$
- $\sqrt[n]{a^m} = a^{\frac{m}{n}}$
- $a^0 = 1$

Example:

Given that $7^{6-x} \div 3^{9+x} = 3^{x-1} \times 7^x$, find the exact value of 21^x.

Sec 4 June 2023 Additional Math Revision II

Example:

Use the substitution $u = 3^x$ to solve the equation $5(3^{2x+1}) - 3^{x+1} = 12$.

Practice Questions

1) Show that $\frac{2^x 3^{x+1} - 2(6^x)}{3^{1-x} 18^{x-3}}$ is always independent of *x*. [3]

2) Find the values of x and y which satisfy the equations [4]

$$2^{x-y} = \sqrt[4]{\frac{1}{16}}$$

$$\frac{5^x}{25^{-y}} = \left(\frac{1}{5}\right)^{-\frac{1}{3}}$$

3) Show that $5^{n+1} + 5^{n+2} + 5^{n+3}$ is divisible by 31 for all positive integer values of n. [3]

Sec 4 June 2023 Additional Math Revision II

Surds

Summary:

- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- Rationalisation

Example:

Express $\frac{\sqrt{2}-5\sqrt{3}}{3\sqrt{2}+\sqrt{3}}$ in the form $\frac{a+b\sqrt{6}}{c}$, where *a*, *b* and *c* are integers.

Sec 4 June 2023 Additional Math Revision II

Practice Question

4) Express $\frac{30+18\sqrt{3}}{2+\sqrt{3}}$ in the form $r + s\sqrt{3}$, where r and s are integers. [3]

5) The volume of a right circular cone is 4π cm^3. The radius of its base is $\left(1 + \sqrt{2}\right)$cm. Find, without the use of a calculator, the height of the cone in the form $\left(a + b\sqrt{2}\right)$ cm, where a and b are integers. [3]

6) Solve $\sqrt{4x + 12} - \sqrt{x + 3} = 2$. [3]

Sec 4 June 2023 Additional Math Revision II

Logarithm

Summary

- $\lg x = \log_{10} x$
- $\ln x = \log_e x$
- $\log_a b = c \Leftrightarrow b = a^c$ (Most important formula)
- $\log_a b + \log_a c = \log_a (bc)$
- $\log_a b - \log_a c = \log_a \left(\frac{b}{c}\right)$
- $\log_a b^c = c \log_a b$
- $\log_a a = 1$
- $\log_a b = \frac{\log_c b}{\log_c a}$

Example: Solve the following equations

a) $\log_2(x^2 - 6) + \log_2 2 = 1 + \log_2 x$

b) $\log_3(2x - 1) = \log_9(2x - 1) + 1$

c) $\log_4 x^2 - 3\log_x 4 = 1$

Sec 4 June 2023 Additional Math Revision II

Practise Question

7) Solve the following equations

a) Solve $\log_x 27 + 2\log_3 x = 7$ [5]

b) $2\log_4(x-6) + \log_2(x+6) = \frac{1}{\log_5 2} + \log_2 x$. [4]

8) Evaluate $\log_a 8 \times \log_{16} a$. [3]

9) Given that $m = \log_3 p$ **,** express the following in terms of *m*.

a) $\log_3 \frac{p^2}{3}$ [2]

b) $\log_p 9$ [2]

10) The population, P of a certain micro-organism present at t hour after initially being observed is given by the formula $P = 160 + 220e^{kt}$, where k is a constant. It took 4 hours to reach a population of 620.

a) Calculate the population of the micro-organisms after 24 hours. [4]

b) Explain why the formula is not suitable to calculate the long-term population of the micro-organisms. [2]