EQUITY
LEARNING PLACE

Sec 4 June 2023 Additional Math Revision

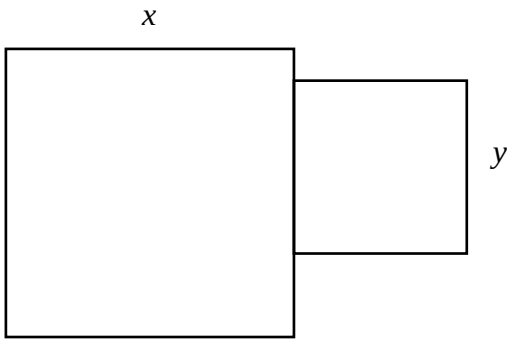
Simultaneous Equation:

Summary:

- Use simultaneous equation to solve when we have 2 unknowns.
- Finding the **intersection** of 2 curves/ lines.
- Elimination method may not work all the time.

Example 1:

A composite figure is formed by two squares of lengths x cm and y cm respectively where x and y are integers.



Given that the area and perimeter of the composite figure is 136 cm^2 and 52 cm respectively, find the value of x and of y.

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Practice Questions:

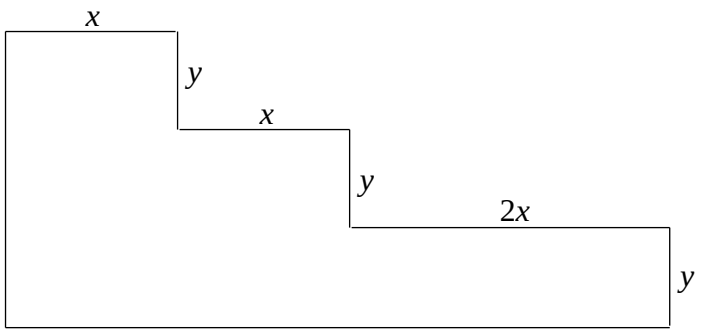
1) Solve the simultaneous equations. [5]

$$\frac{2}{x} + y = 9$$

$$x + \frac{5}{y} = \frac{3}{2}$$

2) The line $3 + 3y - 2x = 0$ meets the curve $x^2 - y^2 = 4x - 1$ at the points A and B. Find the coordinate of A and B. [4]

3) The shape of a ladder is modelled using rectangles in the figure below. The perimeter of the shape is 38 cm and the area enclosed is 52.5 cm^2.



Find all possible values of x and of y. [5]

Quadratic Inequalities:

Summary

- Quadratic inequalities must **draw graph** to solve.

Example 2:

Represent the solution set of $-15 < x^2 - 8x \leq 0$ on a number line and determine if $x = 5$ satisfies the inequality.

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Practice Questions:

4) Find the set of values of x for which $5x^2 + 12x > 3x + 2$. Represent your answer on a number line. [3]

5) Find the range of values of x for which $2x^2 - 8x + 6 > 2x - 2 - x^2$. [3]

6) Find the range of values of x for which [3]

$$1 - x^2 \geq \frac{x^2 + 5x}{-3}$$

Application of Completing the Square

Summary

- **Coefficient** of x must be **1** before completing the square.
- It is meant for **maximum** and **minimum** point of quadratic function.
- It can be used to determine the quadratic function is **positive** or **negative** for all values of x.

Example:

i) Express $3x^2 - 6x + 5$ and $-x^2 - 4x - 3$ in the form $a(x + b)^2 + c$, where a, b and c are constants.

ii) Using your answers from part (i), explain why the two curves $y = 3x^2 - 6x + 5$ and $y = -x^2 - 4x - 3$ will not intersects.

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Practice Questions:

7a) Express $4x^2 - 4x - 19$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [2]

7b) Find the coordinates of the turning point on the graph $y = 4x^2 - 4x - 19$. [1]

7c) Find the exact values of the x – intercepts. [3]

8) Express each of following in the form of $a(x+b)^2 + c$, where a, b and c are constants. [4]

a) $6x^2 - 12x + 9$

b) $-2x^2 - 4x - 2$

c) Using your answers above, explain whether the curves with equations $y = 6x^2 - 12x + 9$ and $y = -2x^2 - 4x - 2$ will intersect each other. [3]

9) Given the curve $y = p(x+q)^2 + r$ is always negative.
State two conditions for p and r. [2]

Discriminant:

Summary:

- Comes from quadratic formula.
- Must have a quadratic equation equal to 0 before you can apply.
- No real roots, $b^2 - 4ac < 0$.
- 1 real root or equal real root, $b^2 - 4ac = 0$.
- 2 real roots, $b^2 - 4ac > 0$.
- Always positive, $b^2 - 4ac < 0$.
- Always negative, $b^2 - 4ac < 0$.

Skills needed,

- Quadratic inequalities
- Simultaneous equation
- Completing the square

Example:

A curve has the equation $2y - \frac{4}{x} = k$ and a line has an equation $\frac{1}{2}x + y = k$, where k is a constant. Find the range of values of k for which the curve and the line do not intersect and represent the solution on a number line. [5]

Example:

Show that the curve $y = -3x^2 + kx + 2$ intersects the line $y = x - 5$ for all real values of k.

[3]

Practise Question

10) Find the range of values of the positive constant k for which the line $y = kx$ lies completely above the curve $y = -16x^2 + (2k-1)x - 1$. [4]

11a) Determine the set of values of m for which the equation $2x^2 + 4x + 2m = 6mx - 2$ has real roots. [4]

11b) Hence, state what can be deduced about the curve $y = 2(x+1)^2$ and the line $y = 6x - 2$. Justify your statement. [2]

12a) Find the range of values of p for which the equation $px^2 + x + p(x+1) = 0$ has real and distinct roots. [3]

12b) State the value(s) of p for which the curve $y = px^2 + x + p(x+1)$ is tangent to the x-axis. [1]

Application of Quadratic Function

Summary:

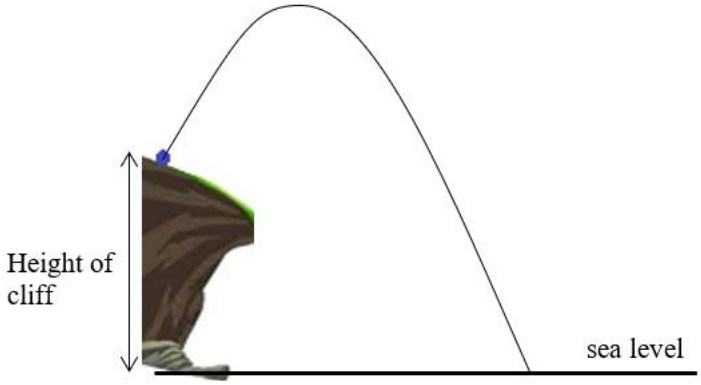
- Involve Quadratic Graph.
- Minimum and Maximum point of Quadratic Graph.
- y-intercept of Quadratic Graph
- x-intercept of Quadratic Graph
- Line of symmetry of Quadratic Graph

Skills needed:

- Completing the Square
- Solving Quadratic Equations.

Example:

A ball is thrown from a cliff overlooking the sea. The vertical height of the ball above sea level, h metres, is given by $h = -8t^2 + 36t + 20$, where t is the time in seconds after the ball is thrown.

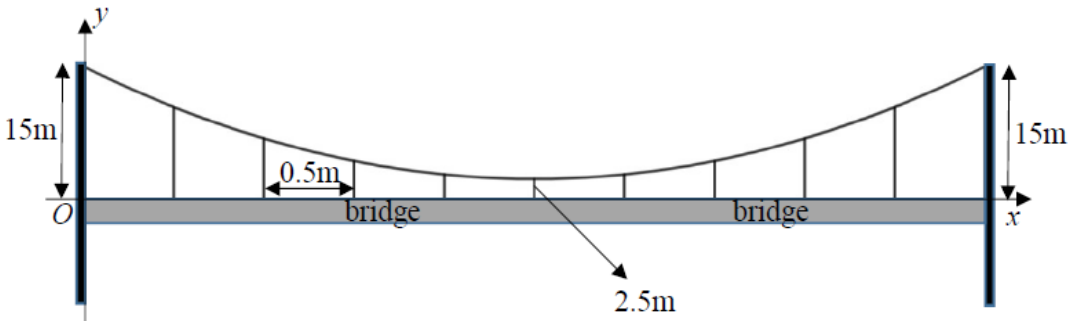


a) Find the height of the cliff.

b) By expressing h in its completed square form, determine whether the ball can reach a height of 65 metres above sea level.

Practice Questions

13) A portion of a suspension bridge has a curved cable hanging between two pillars which can be modelled by a quadratic function. In the model, x m is the horizontal distance from the left pillar and y m is the height of the cable above the bridge. Vertical supporting wires are spread out in 10 equal intervals of 0.5 m apart. At its lowest point, the cable is situated 2.5 m above the bridge.



a) Write down the coordinates of the lowest point. [1]

b) Form a quadratic equation in the form $y = a(x + b)^2 + c$ to model this situation. Hence find the value of a, b and c. [4]

14) A man launched a rocket on a cliff. The motion of the rocket can be modelled by a quadratic function $y = -0.5x^2 + 6x + 182$, where y m is the height of the rocket above the ground and x is the time in seconds, after the rocket is launched into the sky.

a) State the height of the rocket above the ground. [1]

b) Find the height reached by the rocket 2 seconds after it was fired into the sky. [1]

c) Find the maximum height reached by the rocket and the corresponding value of x. [4]

d) The man designed the rocket to be more than 20 seconds in the air. Explain whether the rocket met this criteria. [4]

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Polynomial (Remainder and Factor Theorem)

Summary:

- Remainder Theorem : If f(x) is divided by $(x-a)$, the remainder is f(a).
- Factor Theorem : If f$(a)=0$, $(x-a)$ is a factor of f(x).

Skills needed:

- Remainder and Factor Theorem
- Simultaneous Equation

Example:

Given that $p(x)=(x+1)^3+5(x-1)^2+ax+b$ has a factor of $x-1$ and leaves a remainder of 21 when divided by $x-2$, find the value of a and of b.

Practice Questions

15) The expression $px^3 + qx^2 - 29x - 6$ where p and q are constants has a factor of $x + 3$ and leaves a remainder of -24 when divided by $x - 1$. Find the value of p and q. [4]

16) The function $f(x) = 2x^3 - 10x^2 + ax + b$, where a and b are constants, leaves a remainder of 3 and -3 when divided by $(x + 1)$ and $(x - 2)$ respectively. Find the value of a and b. [4]

17) When a polynomial $f(x)$ is divided by $(x - 1)$, the remainder is 4. When the same polynomial is divided by $(x + 3)$, the remainder is -24. Find the remainder when $f(x)$ is divided by $(x^2 + 2x - 3)$. [5]

Solving Cubic Equations

Summary:

- Guess a factor
- Long division or Compare Coefficient
- Solve Linear and Quadratic Factors.

Skill needed

- Factor Theorem
- Long Division
- Solve Quadratic Equations

Example:

It is given that $x - 4$ is a factor of the polynomial $f(x) = 9x^3 + ax^2 - 23x + b$, where a and b are constants. It leaves a remainder of -20 when it is divided by $x + 1$.

a) Find the values of a and of b. [5]

b) Using the values of a and b found in part (i), solve the equation $f(x) = 0$. [4]

c) Hence, use your answers to part (ii) to solve the equation
$9x^6 + ax^4 - 23x^2 + b = 0$ [2]

Practise Questions

18) The equation of a polynomial is given by $f(x)=3x^3+5x^2+7x-3$.

a) Find the remainder when f(x) is divided by $x + 1$. [1]

b) Show that $3x - 1$ is a factor of f(x). [1]

c) Show that the equation f(x) = 0 has only one real root. [4]

d) Use your answers to parts **(ii)** and **(iii)** to solve the equation

$\frac{3}{2}\left(2^{3y+1}\right)+5(2^{2y})+7(2^{y})=3$. [4]

19) It is given that $f(x) = 2x^3 - 11x^2 + 5x + 18$.

i) Show that $x - 2$ is a factor of f(x). [1]

ii) Given that $2x^3 - 11x^2 + 5x + 18 = (ax^2 + bx + c)(x - 2)$, where a, b and c are constants, find the values of a, b and c. [3]

iii) Hence, solve the equation $f(x) = 0$. [2]

iv) Use the roots of $f(x) = 0$ to solve the equation $2 + 5x^2 + 18x^3 = 11x$. [3]

Partial Fraction

Summary:

- Long Division if Improper Fraction.
- Identities
- $\frac{}{x-a} = \frac{A}{x-a}$
- $\frac{}{(x-a)^2} = \frac{A}{x-a} + \frac{B}{(x-a)^2}$
- $\frac{}{x^2+a^2} = \frac{Ax+B}{x^2+a^2}$

Example:

Express $\frac{2x^3-1}{x^3-x^2}$ in partial fractions. [6]

Practise Questions

20) Express $\frac{5x^2+7x+2}{(2x-1)(x^2+2x)}$ in partial fractions. [5]

21) Express $\frac{5x^3+3x^2+11x+8}{x^3+2x}$ in partial fractions. [6]

22) Express $\frac{7x^2+19x+15}{(x+1)^2(x+2)}$ in partial fractions. [5]

Binomial Theorem

Summary:

- Binomial Theorem on Expansion.

$$(a+b)^n = \binom{n}{0}(a)^0(b)^n + \binom{n}{1}(a)^1(b)^{n-1} + \binom{n}{2}(a)^2(b)^{n-2} + \cdots$$

- General Term.

$$(a+b)^n = \cdots + \binom{n}{r}(a)^r(b)^{n-r} + \cdots$$

- Rewriting $\binom{n}{r}$ into algebraic form.
  Example:

$$\binom{12}{3} = \frac{12 \times 11 \times 10}{1 \times 2 \times 3}$$

Example:

i) Write down the first 3 terms in the expansion of $\left(2 - \frac{x}{3}\right)^6$, in ascending power of x. [3]

ii) Hence, find the term independent of x in the expansion of $\left(2 - \frac{x}{3}\right)^6 \left(\frac{4}{x} - x\right)^2$. [3]

Example:

i) Find the term independent of x in the binomial expansion of $\left(2x^2 + \frac{1}{x}\right)^9$. [3]

ii) Hence, find the coefficient of the term in x^3 in the expansion of

$\left(2x^2 + \frac{1}{x}\right)^9 \left(2x^3 - \frac{1}{12x^6}\right)$. [3]

Example:

a) Write down and simplify, in descending powers of *x*, the first three terms in the expansion of $\left(x^5 + \frac{2}{x^6}\right)^n$, where $n > 0$. [2]

b) When the third term of the expansion is divided by the second term, $\frac{8}{x^{11}}$ is obtained. Show that $n = 9$. [2]

c) Using the value of *n* found in **(ii)**, without expanding $\left(x^5 + \frac{2}{x^6}\right)^n$, show that there is no constant term in the expansion. [3]

Practise Questions

23) In the binomial expansion of $\left(x^2 - \frac{1}{x}\right)^{12}$, find

a) the term containing x^9. [4]

b) the term independent of x. [2]

24a) Write down and simplify the first three terms in the expansion, in ascending powers of *x*, of $(2-3x)^5$. [3]

24b) Given that the expansion of $(p+qx)(2-3x)^5$ up to the term in x^2 is $8+rx+1680x^2$, find the value of *p,* or *q* and of *r*. [4]

25) The first 3 terms in the expansion, in ascending powers of *x*, of $(1-3x)^n$, is $1-21x+ax^2$, where *a* is a constant and *n* is a positive integer greater than 2.
Find the value of *n* and of *a*. [4]

25b) Using your values of *n* and *a*, find the coefficient of x^2 in the expansion of $(2+x)^2(1-3x)^n$. [2]