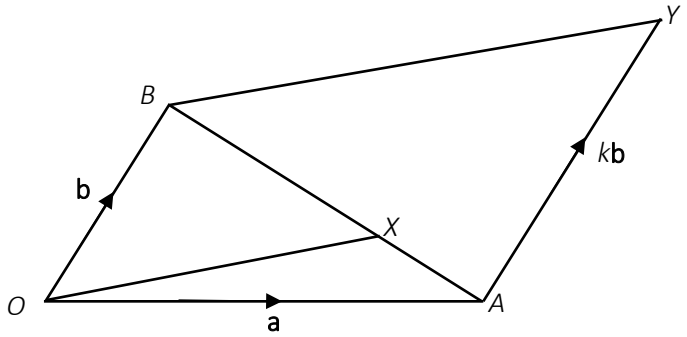
EQUITY
LEARNING PLACE

Elementary Math Topical (Vector)

Question 1:

In the diagram, $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{AY} = k\mathbf{b}$. X lies on the line AB such that $\overrightarrow{AX} = \frac{1}{3}\overrightarrow{AB}$



Express the following vectors, as simply as possible, in terms of ***a*** and ***b***, find

a) $\overrightarrow{AX}$

b) $\overrightarrow{OX}$

c) Express $\overrightarrow{BY}$ in terms of k, **a** and **b**

d) Given that $\overrightarrow{OX}$ is parallel to $\overrightarrow{BY}$, find the value of k.

e) The line OX when produced, meets AY at Z. Find ratio of $AZ : OB$.

f) $\dfrac{\text{area of } \Delta OAX}{\text{area of } \Delta OBX}$

g) $\dfrac{\text{area of } \Delta OAX}{\text{area of quadrilateral OAYB}}$

a)

$$\overrightarrow{AX} = \frac{1}{3}\overrightarrow{AB}$$

$$= \frac{1}{3}\left(\overrightarrow{AO} + \overrightarrow{OB}\right)$$

$$= \frac{1}{3}\left(-\underset{\sim}{a} + \underset{\sim}{b}\right)$$

$$= -\frac{1}{3}\underset{\sim}{a} + \frac{1}{3}\underset{\sim}{b}$$

b)

$$\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX}$$

$$= \underset{\sim}{a} - \frac{1}{3}\underset{\sim}{a} + \frac{1}{3}\underset{\sim}{b}$$

$$= \frac{2}{3}\underset{\sim}{a} + \frac{1}{3}\underset{\sim}{b}$$

c)

$$\overrightarrow{BY} = \overrightarrow{BA} + \overrightarrow{AY}$$

$$= -\underset{\sim}{b} + \underset{\sim}{a} + k\underset{\sim}{b}$$

$$= \underset{\sim}{a} + (-1 + k)\underset{\sim}{b}$$

d)

$$\frac{2}{3}\underset{\sim}{a} + \frac{1}{3}\underset{\sim}{b} = h\left(\underset{\sim}{a} + (-1 + k)\underset{\sim}{b}\right)$$

$$\frac{2}{3}\underset{\sim}{a} + \frac{1}{3}\underset{\sim}{b} = h\underset{\sim}{a} + (-1 + k)h\underset{\sim}{b}$$

By comparison

$$h = \frac{2}{3}$$

$$\frac{1}{3} = (-1 + k)\left(\frac{2}{3}\right)$$

$$1 = -2 + 2k$$

$$k = \frac{3}{2}$$

e)

$$\overrightarrow{AZ} = \frac{1}{2}\underset{\sim}{b}$$

$$AZ : OB = 1 : 2$$

f)

$$\frac{\text{Area of } OAX}{\text{Area of } OBX} = \frac{\frac{1}{2} \times AX \times h}{\frac{1}{2} \times BX \times h} = \frac{AX}{BX} = \frac{1}{2}$$

g)

$$\frac{\text{Area of } OBA}{\text{Area of } ABY} = \frac{\frac{1}{2} \times OB \times h}{\frac{1}{2} \times AY \times h} = \frac{OB}{AY} = \frac{2}{3}$$

$$\frac{\text{Area of } OAX}{\text{Area of } OAYB} = \frac{\text{Area of } OAX}{\text{Area of } AOB} \times \frac{\text{Area of } AOB}{\text{Area of } OAYB} = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$$

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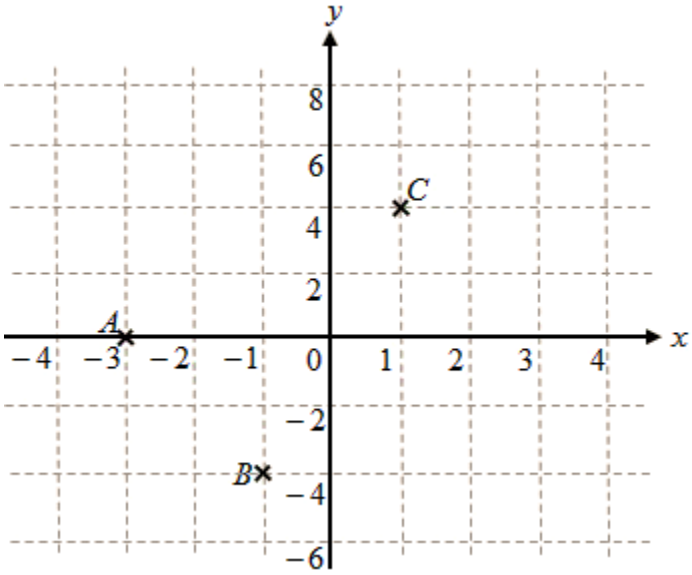
Question 2:

The diagram shows three points $A(-3,0)$, $B(-1,-4)$ and $C(1,4)$.



a) Express $\overrightarrow{AC}$ as column vector.

b) Given that $\overrightarrow{OS} = \begin{pmatrix} 2 \\ a \end{pmatrix}$ is parallel to $\overrightarrow{AC}$, find the value of *a*.

c) Find $|2\overrightarrow{AC}|$

d) Find the position vector of point *D* given that *ABCD* is a parallelogram

a)

$$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

b)

$$\begin{pmatrix} 2 \\ a \end{pmatrix} = k \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

$$4k = 2$$

$$k = \frac{1}{2}$$

$$a = 2$$

c)

$$\left|2\overrightarrow{AC}\right| = 2\sqrt{4^2 + 4^2} = 11.3 \text{ units}$$

d)

$$\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD} = \overrightarrow{OB} + \overrightarrow{AC} = \begin{pmatrix}-1\\-4\end{pmatrix} + \begin{pmatrix}4\\4\end{pmatrix} = \begin{pmatrix}3\\0\end{pmatrix}$$

Question 3:

Given that $\overrightarrow{OM} = \begin{pmatrix}3\\-5\end{pmatrix}$, $\overrightarrow{ON} = \begin{pmatrix}-5\\-7\end{pmatrix}$ where *O* is the origin and *P* is a point on the *y*-axis such that $\overrightarrow{MP} = \begin{pmatrix}-3\\10\end{pmatrix}$,

a) express $\overrightarrow{NM}$ as a column vector

b) find the coordinates where the line *MP* crosses the *x*-axis.

a)

$$\overrightarrow{NM} = \overrightarrow{NO} + \overrightarrow{OM} = \begin{pmatrix}5\\7\end{pmatrix} + \begin{pmatrix}3\\-5\end{pmatrix} = \begin{pmatrix}8\\2\end{pmatrix}$$

b) Let the point on $x-$ axis be A

$$\overrightarrow{OA} = \begin{pmatrix}x\\0\end{pmatrix}$$

$$\overrightarrow{MA} = \overrightarrow{MO} + \overrightarrow{OA} = \begin{pmatrix}-3\\5\end{pmatrix} + \begin{pmatrix}x\\0\end{pmatrix} = \begin{pmatrix}x-3\\5\end{pmatrix}$$

Since MA // MP

$$\begin{pmatrix}x-3\\5\end{pmatrix} = k\begin{pmatrix}-3\\10\end{pmatrix}$$

$$5 = 10k$$

$$k = \frac{1}{2}$$

$$x - 3 = -\frac{3}{2}$$

$$x = \frac{3}{2}\ , \therefore \left(\frac{3}{2}, 0\right)$$

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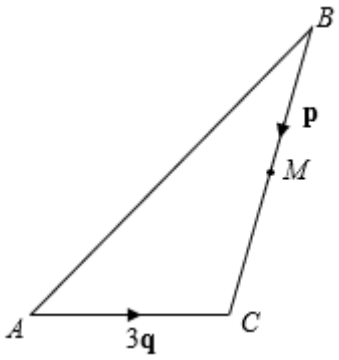
Question 4:

In triangle *ABC* below, *M* is the midpoint of *BC*. It is given that $\overrightarrow{BC}$= **p** and $\overrightarrow{AC}$= 3**q**.



a) Find $\overrightarrow{AM}$ in terms of **p** and **q**.

X is a point outside triangle *ABC* such that $\overrightarrow{BX}$ = *k***q**, where *k* is a positive integer, and $\overrightarrow{CX}$ is parallel to $\overrightarrow{AM}$.

b) Express $\overrightarrow{CX}$ in terms of *k*, **p** and **q**.

c) Find the value of k

Y is a point on *AM* such that 5*AY* = 2*AM*. *Z* is a point on *AB* such that *AZ* : *AB* = 1 : 4. Express, in terms of **p** and **q**,

d) $\overrightarrow{CY}$

e) $\overrightarrow{CZ}$

a)

$$\overrightarrow{AM} = \overrightarrow{AC} + \overrightarrow{CM} = 3\underset{\sim}{q} - \frac{1}{2}\underset{\sim}{p}$$

b)

$$\overrightarrow{CX} = \overrightarrow{CB} + \overrightarrow{BX} = -\underset{\sim}{p} + k\underset{\sim}{q}$$

c) Since CX // AM

$$-\underset{\sim}{p} + k\underset{\sim}{q} = h\left(3\underset{\sim}{q} - \frac{1}{2}\underset{\sim}{p}\right)$$

By comparison

$$-1 = -\frac{1}{2}h$$

$$h = 2$$

$$k = 6$$

d)

$$\overrightarrow{CY} = \overrightarrow{CA} + \overrightarrow{AY} = -3\underset{\sim}{q} + \frac{2}{5}\left(3\underset{\sim}{q} - \frac{1}{2}\underset{\sim}{p}\right) = -\frac{9}{5}\underset{\sim}{q} - \frac{1}{5}\underset{\sim}{p}$$

e)

$$\overrightarrow{CZ} = \overrightarrow{CA} + \overrightarrow{AZ} = -3\underset{\sim}{q} + \frac{1}{4}\left(3\underset{\sim}{q} - \underset{\sim}{p}\right) = -\frac{11}{4}\underset{\sim}{q} - \frac{1}{4}\underset{\sim}{p}$$

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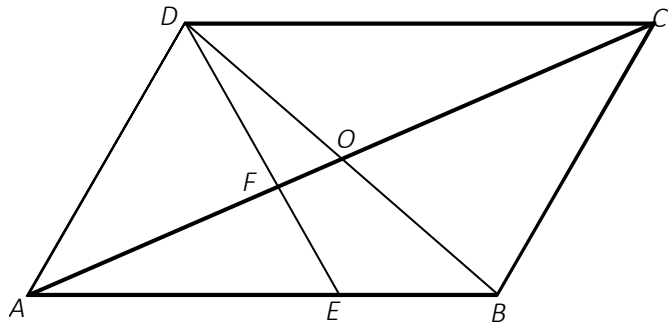
Question 5:

ABCD is a parallelogram whose diagonals, *AC* and *BD*, intersect at *O*. *E* is a point on *AB* such that *AE* = 2*EB*. *DE* intersects *AC* at *F*.



Given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, express the following vectors in terms of **a** and/or **b**, giving each of your answers in its simplest form,

a) $\overrightarrow{AC}$

b) $\overrightarrow{CD}$

c) $\overrightarrow{DA}$

d) Show that $\overrightarrow{DE} = \frac{1}{3}(\mathbf{a} + 5\mathbf{b})$

e) Given that $\overrightarrow{FA} = \frac{4}{5}\overrightarrow{OA}$, show that *D*, *E* and *F* lie on the same straight line.

Find the numerical value of

f) $\frac{\text{Area of } \Delta AEF}{\text{Area of } \Delta CDF}$

g) $\frac{\text{Area of } \Delta DOF}{\text{Area of } \Delta AEF}$

a)

$$\overrightarrow{AC} = -2\underset{\sim}{a}$$

b)

$$\overrightarrow{CD} = \overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA} = -\underset{\sim}{b} + \underset{\sim}{a}$$

c)

$$\overrightarrow{DA} = \overrightarrow{DC} + \overrightarrow{CA} = \underset{\sim}{b} - \underset{\sim}{a} + 2\underset{\sim}{a} = \underset{\sim}{a} + \underset{\sim}{b}$$

d)

$$\overrightarrow{DE} = \overrightarrow{DA} + \overrightarrow{AE} = \underset{\sim}{a} + \underset{\sim}{b} + \frac{2}{3}\left(\underset{\sim}{b} - \underset{\sim}{a}\right) = \frac{1}{3}\underset{\sim}{a} + \frac{5}{3}\underset{\sim}{b} = \frac{1}{3}\left(\underset{\sim}{a} + 5\underset{\sim}{b}\right)$$

e)

$$\overrightarrow{DF} = \overrightarrow{DA} + \overrightarrow{AF} = \underset{\sim}{a} + \underset{\sim}{b} - \frac{4}{5}\left(\underset{\sim}{a}\right) = \frac{1}{5}\left(\underset{\sim}{a} + 5\underset{\sim}{b}\right)$$

$$= \frac{3}{5}\left[\frac{1}{3}\left(\underset{\sim}{a} + 5\underset{\sim}{b}\right)\right]$$

$$= \frac{3}{5}\overrightarrow{DE}$$

$\therefore DE$ is parallel to DE and D is a common point so D, E and F form a straight line.

f) AEF and CDF are similar triangle

$$\frac{\text{Area AEF}}{\text{Area CDF}} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

g)

$$\frac{\text{Area DOF}}{\text{Area AEF}}$$

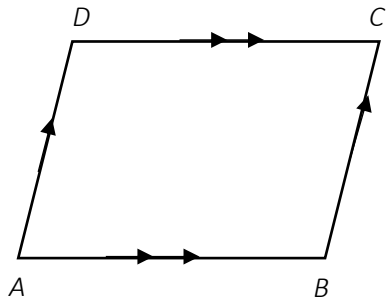
$$= \frac{\text{Area DOF}}{\text{Area AFD}} \times \frac{\text{Area AFD}}{\text{Area AEF}}$$

$$= \frac{\frac{1}{2} \times OF \times h}{\frac{1}{2} \times AF \times h} \times \frac{\frac{1}{2} \times FD \times H}{\frac{1}{2} \times FE \times H}$$

$$= \frac{1}{4} \times \frac{3}{2} = \frac{3}{8}$$

Question 6:

The position vectors of points *A*, *B* and *C* are $\overrightarrow{OA} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$ respectively. A point *D* is such that *ABCD* is a parallelogram.



a) Find $\overrightarrow{AB}$.

b) Hence, find the coordinates of *D*.

The position of point E is $\overrightarrow{OE} = \begin{pmatrix} 9 \\ e \end{pmatrix}$.

c) Find $|\overrightarrow{CE}|$ in terms of *e*.

d) Given that $|\overrightarrow{CE}| = 2|\overrightarrow{AC}|$, find the possible values of *e*.

a)

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$$

b)

$$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD} = \overrightarrow{OC} + \overrightarrow{BA} = \begin{pmatrix} 3 \\ -3 \end{pmatrix} + \begin{pmatrix} -3 \\ -7 \end{pmatrix} = \begin{pmatrix} 0 \\ -10 \end{pmatrix}$$

c)

$$\overrightarrow{CE} = \overrightarrow{CO} + \overrightarrow{OE} = \begin{pmatrix} -3 \\ 3 \end{pmatrix} + \begin{pmatrix} 9 \\ e \end{pmatrix} = \begin{pmatrix} 6 \\ 3 + e \end{pmatrix}$$

$$|\overrightarrow{CE}| = \sqrt{(6)^2 + (3 + e)^2} = \sqrt{36 + (3 + e)^2}$$

d)

$$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = \begin{pmatrix}2\\3\end{pmatrix} + \begin{pmatrix}3\\-3\end{pmatrix} = \begin{pmatrix}5\\0\end{pmatrix}$$

$$|\overrightarrow{CE}| = 2|\overrightarrow{AC}|$$

$$\sqrt{36 + (3+e)^2} = 2(5)$$

$$36 + 9 + 6e + e^2 = 100$$

$$e^2 + 6e - 55 = 0$$

$$(e + 11)(e - 6) = 0$$

$$e = -11 \text{ or } e = 6$$

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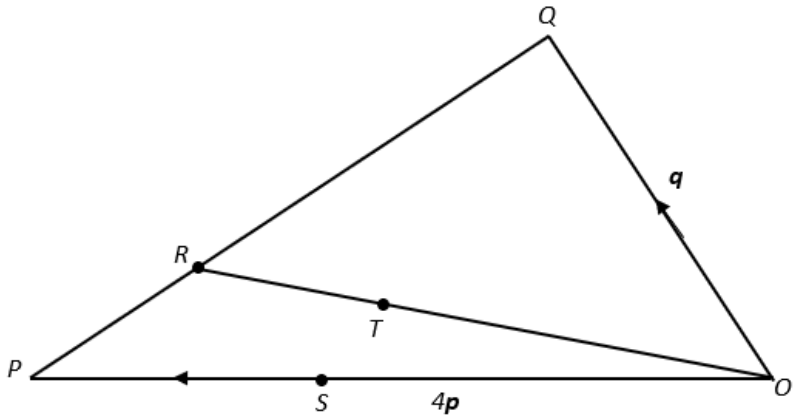
Question 7:

In the diagram, *OPQ* is a triangle and *R*, *S* and *T* are points on *PQ*, *OP* and *OR* respectively. It is given that *OS* : *OP* = 4 : 7, *OT* : *OR* = 2 : 3, *PR* :*PQ* = 1 : 3, $\overrightarrow{OS}$ = 4**p** and $\overrightarrow{OQ}$ = **q**.



Express the following vectors in terms of **p** and **q**.

a) $\overrightarrow{QS}$

b) $\overrightarrow{OR}$

c) $\overrightarrow{OT}$

d) $\overrightarrow{QT}$

e) State two facts about the vectors $\overrightarrow{QT}$ and $\overrightarrow{QS}$.

Find the ratio of the areas of

f) ΔOQT and ΔOQS

g) ΔOQS and ΔOQP.

a)

$$\overrightarrow{QS} = \overrightarrow{QO} + \overrightarrow{OS} = -\underset{\sim}{q} + 4\underset{\sim}{p}$$

b)

$$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ} = -7\underset{\sim}{p} + \underset{\sim}{q}$$

$$\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR} = 7\underset{\sim}{p} + \frac{1}{3}\left(-7\underset{\sim}{p} + \underset{\sim}{q}\right) = \frac{14}{3}\underset{\sim}{p} + \frac{1}{3}\underset{\sim}{q}$$

c)

$$\overrightarrow{OT} = \frac{2}{3}\overrightarrow{OR} = \frac{2}{3}\left(\frac{14}{3}\underset{\sim}{p} + \frac{1}{3}\underset{\sim}{q}\right) = \frac{28}{9}\underset{\sim}{p} + \frac{2}{9}\underset{\sim}{q}$$

d)

$$\overrightarrow{QT} = \overrightarrow{QO} + \overrightarrow{OT} = -\underset{\sim}{q} + \frac{28}{9}\underset{\sim}{p} + \frac{2}{9}\underset{\sim}{q} = \frac{28}{9}\underset{\sim}{p} - \frac{7}{9}\underset{\sim}{q}$$

e)

$$\overrightarrow{QT} = \frac{28}{9}\underset{\sim}{p} - \frac{7}{9}\underset{\sim}{q}$$

$$= \frac{7}{9}\left(-\underset{\sim}{q} + 4\underset{\sim}{p}\right)$$

$$= \frac{7}{9}\overrightarrow{QS}$$

QT is parallel to QS and $\overrightarrow{QT} = \frac{7}{9}\overrightarrow{QS}$ or they form a straight line.

f)

$$\text{Area of } OQT : \text{Area of } OQS$$

$$\frac{1}{2} \times QT \times h : \frac{1}{2} \times QS \times h$$

$$QT : QS$$

$$7 : 9$$

g)

$$\text{Area of } OQS : \text{Area of } OQP$$

$$\frac{1}{2} \times OS \times h : \frac{1}{2} \times OP \times h$$

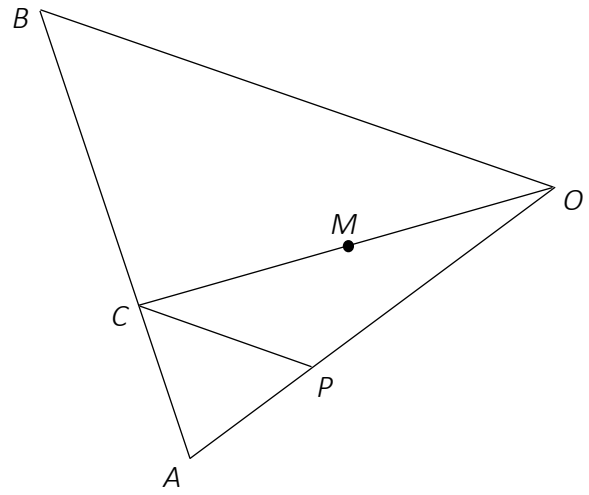
$$OS : OP$$

$$4 : 7$$

Elementary Math Topical (Vector)

Question 8:

OAB is a triangle. $\overrightarrow{OA}$ = 3**a** and $\overrightarrow{OB}$ = 2**b**. $\overrightarrow{OP} = \frac{2}{3}\overrightarrow{OA}$ and $\overrightarrow{AC} = \frac{1}{3}\overrightarrow{AB}$. *M* is a point on *OC* such that *OM* = *MC*.



Express, as simply as possible, in terms of **a** and/or **b**,

a) $\overrightarrow{AB}$

b) $\overrightarrow{PC}$

c) $\overrightarrow{OC}$

d) Explain why $\overrightarrow{PC}$ is parallel to $\overrightarrow{OB}$.

e) Show that triangle ACP and ABO are similar.

f) Write down the ratio of the area of triangle ACP to the area of the quadrilateral CPOB.

g) X is a point on OB such that OB = 3OX. Show that P, M and X are collinear.

a)

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -3\underset{\sim}{a} + 2\underset{\sim}{b}$$

b)

$$\overrightarrow{PC} = \overrightarrow{PA} + \overrightarrow{AC} = \underset{\sim}{a} + \frac{1}{3}\left(-3\underset{\sim}{a} + 2\underset{\sim}{b}\right) = \frac{2}{3}\underset{\sim}{b}$$

c)

$$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = 3\underset{\sim}{a} + \frac{1}{3}\left(-3\underset{\sim}{a} + 2\underset{\sim}{b}\right) = 2\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{b}$$

d)

$$\overrightarrow{PC} = \frac{2}{3}\underset{\sim}{b} = \frac{1}{3}\left(2\underset{\sim}{b}\right) = \frac{1}{3}\overrightarrow{OB}$$

$$\therefore PC \text{ is parallel to } OB$$

e)

$$\angle APC = \angle AOB \text{ (corres. angle)}$$

$$\angle ACP = \angle ABO \text{ (corres. angle)}$$

By AA similarity test, triangle ACP and ABO are similar.

f)

$$\text{Area } ACP : \text{Area } ABO$$

$$1^2 : 3^2$$

$$1 : 9$$

$$\text{Area } ACP : \text{Area } CPOB$$

$$1 : 8$$

g)

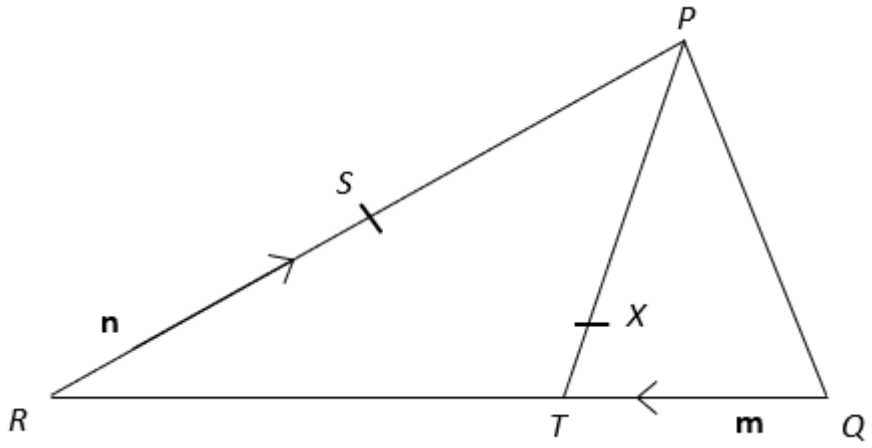
$$\overrightarrow{PX} = \overrightarrow{PO} + \overrightarrow{OX} = -2\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{b}$$

$$\overrightarrow{PM} = \overrightarrow{PO} + \overrightarrow{OM} = -2\underset{\sim}{a} + \frac{1}{2}\left(2\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{b}\right) = -\underset{\sim}{a} + \frac{1}{3}\underset{\sim}{b}$$

$$\overrightarrow{PX} = -2\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{b} = 2\left(-\underset{\sim}{a} + \frac{1}{3}\underset{\sim}{b}\right) = 2\,\overrightarrow{PM}$$

Since PX is parallel to PM and P is a common point, P, Q and R are collinear.

Question 9:



In the diagram, *QR* = 4*QT* and *TP* = 5*TX*. *S* is the midpoint of *PR*. $\overrightarrow{QT} = \mathbf{m}$ and $\overrightarrow{RS} = \mathbf{n}$.

Express, in terms of **m** and/or **n**,

a) $\overrightarrow{TR}$

b) $\overrightarrow{TP}$

c) $\overrightarrow{TX}$

d) $\overrightarrow{QS}$

e) Show that $\overrightarrow{QX} = \frac{2}{5}(4\mathbf{m} + \mathbf{n})$.

f) Write down one fact you can deduce about the points *Q*, *X* and *S*.

Determine the value of

g) $\frac{QX}{QS}$,

h) $\frac{\text{Area of } \Delta PQX}{\text{Area of } \Delta PQT}$

i) $\frac{\text{Area of } \Delta PQX}{\text{Area of } \Delta PQR}$

a)

$$\overrightarrow{TR} = 3\underset{\sim}{m}$$

b)

$$\overrightarrow{TP} = \overrightarrow{TR} + \overrightarrow{RP} = 3\underset{\sim}{m} + 2\underset{\sim}{n}$$

c)

$$\overrightarrow{TX} = \frac{1}{5}\overrightarrow{TP} = \frac{3}{5}\underset{\sim}{m} + \frac{2}{5}\underset{\sim}{n}$$

d)

$$\overrightarrow{QS} = \overrightarrow{QR} + \overrightarrow{RS} = 4\underset{\sim}{m} + \underset{\sim}{n}$$

e)

$$\overrightarrow{QX} = \overrightarrow{QT} + \overrightarrow{TX} = \underset{\sim}{m} + \frac{3}{5}\underset{\sim}{m} + \frac{2}{5}\underset{\sim}{n} = \frac{8}{5}\underset{\sim}{m} + \frac{2}{5}\underset{\sim}{n}$$

f)

$$\overrightarrow{QX} = \frac{8}{5}\underset{\sim}{m} + \frac{2}{5}\underset{\sim}{n} = \frac{2}{5}\left(4\underset{\sim}{m} + \underset{\sim}{n}\right) = \frac{2}{5}\overrightarrow{QS}$$

Q, X and S are collinear.

g)

$$\frac{QX}{QS} = \frac{2}{5}$$

h)

$$\frac{(\text{Area } PQX)}{\text{Area } PQT} = \frac{\frac{1}{2} \times PX \times h}{\frac{1}{2} \times PT \times h} = \frac{4}{5}$$

i)

$$\frac{\text{Area of } PQX}{\text{Area of } PQR} = \frac{\text{Area of } PQX}{\text{Area of } PQT} \times \frac{\text{Area of } PQT}{\text{Area of } PQR}$$

$$= \frac{4}{5} \times \frac{1}{4} = \frac{1}{5}$$

Question 10:

$\overrightarrow{AB} = \begin{pmatrix} 9 \\ -12 \end{pmatrix}$

a) Find $\left|\overrightarrow{AB}\right|$.

b) *D* is the point (–2, 5). $\overrightarrow{DC} = 2\overrightarrow{AB}$. Find the coordinates of *C*.

c) What type of quadrilateral is *ABCD*?

a)

$$\left|\overrightarrow{AB}\right| = \sqrt{9^2 + 12^2} = 15$$

b)

$$\overrightarrow{DC} = \begin{pmatrix} 18 \\ -24 \end{pmatrix}$$

$$\overrightarrow{OC} = \begin{pmatrix} 18 \\ -24 \end{pmatrix} + \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 16 \\ -19 \end{pmatrix}$$

$$C(16, -19)$$

c) Trapezium

Question 11:

XYZ is a triangle.

$\overrightarrow{XY} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$, $\overrightarrow{XZ} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

a) Calculate the length *YZ*.

b) Coordinates of *Y* is $(3,4)$. Find the coordinates of *X*.

a)

$$\overrightarrow{YZ} = \overrightarrow{YX} + \overrightarrow{XZ} = \begin{pmatrix} -2 \\ -2 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$$

$$\left|\overrightarrow{YZ}\right| = \sqrt{4^2 + 1^2} = \sqrt{17}$$

b)

$$\overrightarrow{OX} = \overrightarrow{OY} + \overrightarrow{YX} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Question 12:

Given that $\overrightarrow{OP}=\begin{pmatrix}7\\-2\end{pmatrix}$, $\overrightarrow{OQ}=\begin{pmatrix}4\\6\end{pmatrix}$ and $\overrightarrow{OR}=\begin{pmatrix}-2\\k\end{pmatrix}$. Find

a) $\left|\overrightarrow{PQ}\right|$

b) the value of k if O, P and R lie on the same straight line.

a)

$$\overrightarrow{PQ}=\overrightarrow{PO}+\overrightarrow{OQ}=\begin{pmatrix}-7\\2\end{pmatrix}+\begin{pmatrix}4\\6\end{pmatrix}=\begin{pmatrix}-3\\8\end{pmatrix}$$

$$\left|\overrightarrow{PQ}\right|=\sqrt{3^2+8^2}=\sqrt{73}$$

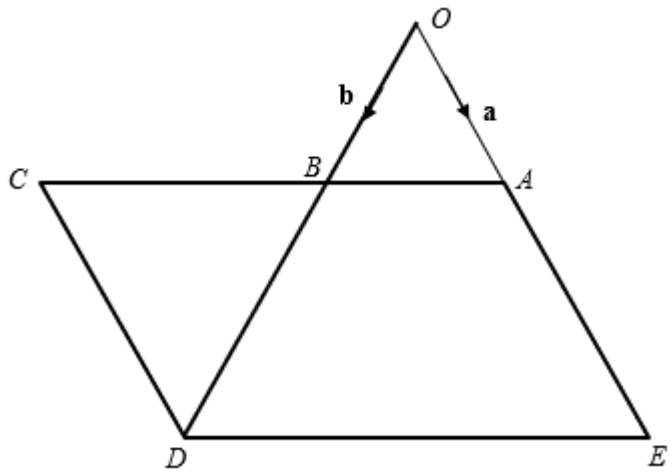
b)

$$\begin{pmatrix}7\\-2\end{pmatrix}=h\begin{pmatrix}-2\\k\end{pmatrix}$$

$$h=-\frac{7}{2}$$

$$k=-2\div-\frac{7}{2}=\frac{4}{7}$$

Question 13:



ACDE is a parallelogram. The point *B* lie on *CA*. The line *DB* produced meets *EA* produced at the point *O* such that $\frac{OB}{BD}=\frac{OA}{AE}=\frac{2}{3}$. It is given that $\overrightarrow{OA}$ = **a** and $\overrightarrow{OB}$ = **b**

Express, as simply as possible in terms of **a** and/or **b**

a) $\overrightarrow{BA}$

b) $\overrightarrow{DE}$

c) $\overrightarrow{EC}$

d) Show that triangle *AOB* and triangle *CDB* are similar.

Find, in its simplest form,

e) $\frac{\text{Area of triangle } OAB}{\text{Area of triangle } OED}$

f) $\frac{\text{Area of triangle } OAB}{\text{Area of triangle } OBE}$

g) $\frac{\text{Area of triangle } BCD}{\text{Area of quadrilateral } ABDE}$

a)

$$\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA} = -\underset{\sim}{b} + \underset{\sim}{a}$$

b)

$$\overrightarrow{DE} = \frac{5}{2}\overrightarrow{BA} = -\frac{5}{2}\underset{\sim}{b} + \frac{5}{2}\underset{\sim}{a}$$

c)

$$\overrightarrow{EC} = \overrightarrow{ED} + \overrightarrow{DC} = \frac{5}{2}\underset{\sim}{b} - \frac{5}{2}\underset{\sim}{a} - \frac{3}{2}\underset{\sim}{a} = \frac{5}{2}\underset{\sim}{b} - 4\underset{\sim}{a}$$

d)

$$\angle AOB = \angle CDB \text{ (alt angle)}$$

$$\angle OAB = \angle DCB \text{ (alt angle)}$$

By AA similarity test, triangle AOB and CDB are similar.

e)

$$\frac{\text{Area of } OAB}{\text{Area of } OED} = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

f)

$$\frac{\text{Area of } OAB}{\text{Area of } OBE} = \frac{\frac{1}{2} \times OA \times h}{\frac{1}{2} \times OE \times h} = \frac{2}{5}$$

g)

$$\frac{\text{Area of } BCD}{\text{Area of } ABDE} = \frac{\text{Area of } BCD}{\text{Area of } OAB} \times \frac{\text{Area of } OAB}{\text{Area of } ABDE}$$

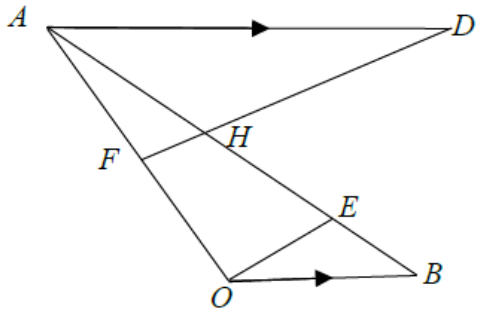
$$= \left(\frac{3}{2}\right)^2 \times \frac{4}{21} = \frac{3}{7}$$

Question 14:

The position vectors A and B, relative to O, are $\mathbf{a}$ and $\mathbf{b}$ respectively.

AD is parallel to OB such that $2AD = 3OB$ and E is a point on AB such that $AE : EB = 3 : 1$.

F is the midpoint of OA.



Express, as simply as possible, in terms of $\mathbf{a}$ and/ or $\mathbf{b}$,

a) $\overrightarrow{OE}$

b) $\overrightarrow{FD}$

c) Write down 2 facts about OE and FD.

d) If FD meets AB at a point H, and the area of OBE is 20 units^2, find the area of AFH.

a)

$$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE} = \underset{\sim}{a} + \frac{3}{4}\overrightarrow{AB} = \underset{\sim}{a} + \frac{3}{4}\left(\underset{\sim}{b} - \underset{\sim}{a}\right) = \frac{1}{4}\underset{\sim}{a} + \frac{3}{4}\underset{\sim}{b}$$

b)

$$\overrightarrow{FD} = \overrightarrow{FA} + \overrightarrow{AD} = \frac{1}{2}\underset{\sim}{a} + \frac{3}{2}\underset{\sim}{b}$$

c)

$$\overrightarrow{OE} = \frac{1}{4}\underset{\sim}{a} + \frac{3}{4}\underset{\sim}{b} = \frac{1}{2}\left(\frac{1}{2}\underset{\sim}{a} + \frac{3}{2}\underset{\sim}{b}\right) = \frac{1}{2}\overrightarrow{FD}$$

OE is parallel to FD

FD is twice the length of OE

d)

$$\frac{\text{Area of } AFH}{\text{Area of } OBE} = \frac{\text{Area of } AFH}{\text{Area of } AOE} \times \frac{\text{Area of } AOE}{\text{Area of } OBE}$$

$$= \left(\frac{1}{2}\right)^2 \times \frac{3}{1} = \frac{3}{4}$$

$$\text{Area of } AFH = \frac{3}{4} \times 20 = 15 \text{ units}^2$$

Question 15:

It is given that $\overrightarrow{AC} = \begin{pmatrix} 8 \\ -3 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} -10 \\ 5 \end{pmatrix}$. *B* is a point on *OC* such that *OB : BC* = 2:3. *A*, *B* and *C* are three points on level ground.

a) Find the coordinates of *A*.

b) Find $\overrightarrow{OB}$

c) Evaluate $\left|\overrightarrow{AB}\right|$

d) Given that $\begin{pmatrix} 2 \\ p \end{pmatrix}$ is parallel to $\overrightarrow{AC}$, find the value of *p*.

a)

$$\overrightarrow{OA} = \overrightarrow{OC} + \overrightarrow{CA} = \begin{pmatrix} -10 \\ 5 \end{pmatrix} + \begin{pmatrix} -8 \\ 3 \end{pmatrix} = \begin{pmatrix} -18 \\ 8 \end{pmatrix}$$

b)

$$\overrightarrow{OB} = \overrightarrow{OC} + \overrightarrow{CB}$$

$$\overrightarrow{OB} = \overrightarrow{OC} + \frac{3}{2}\overrightarrow{BO}$$

$$\frac{5}{2}\overrightarrow{OB} = \begin{pmatrix} -10 \\ 5 \end{pmatrix}$$

$$\overrightarrow{OB} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$$

c)

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \begin{pmatrix} 18 \\ -8 \end{pmatrix} + \begin{pmatrix} -4 \\ 2 \end{pmatrix} = \begin{pmatrix} 14 \\ -6 \end{pmatrix}$$

$$\left|\overrightarrow{AB}\right| = \sqrt{14^2 + 6^2} = 15.2 \text{ units}$$

d)

$$\begin{pmatrix} 2 \\ p \end{pmatrix} = k \begin{pmatrix} 8 \\ -3 \end{pmatrix}$$

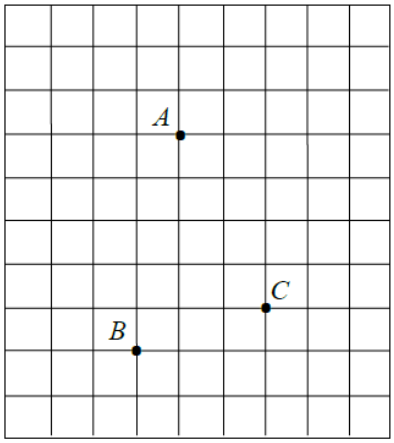
$$k = \frac{1}{4}$$

$$p = -\frac{3}{4}$$

Elementary Math Topical (Vector)

Question 16:

The grid below shows the positions of the points A, B and C where $\overrightarrow{BC} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$



a) Express $\overrightarrow{AC}$ as a column vector.

b) D is a point such that $ABCD$ is a parallelogram. Find the value of $\left|\overrightarrow{CD}\right|$.

a)

$$\overrightarrow{AC} = \begin{pmatrix} 6 \\ -8 \end{pmatrix}$$

b)

$$\overrightarrow{CD} = \overrightarrow{BA} = \begin{pmatrix} 3 \\ 15 \end{pmatrix}$$

$$\left|\overrightarrow{CD}\right| = \sqrt{3^2 + 15^2} = 15.3 \text{ units}$$

Question 17:

OPQR is a parallelogram such that $\overrightarrow{PQ} = \begin{pmatrix}2\\4\end{pmatrix}$ and P is the point (3,2).

a) Express $\overrightarrow{RP}$ as a column vector

b) The point J lies on $\overrightarrow{RP}$ produced such that $\overrightarrow{PJ} = m\overrightarrow{RP}$. Show that $\overrightarrow{OJ} = \begin{pmatrix}3+m\\2-2m\end{pmatrix}$

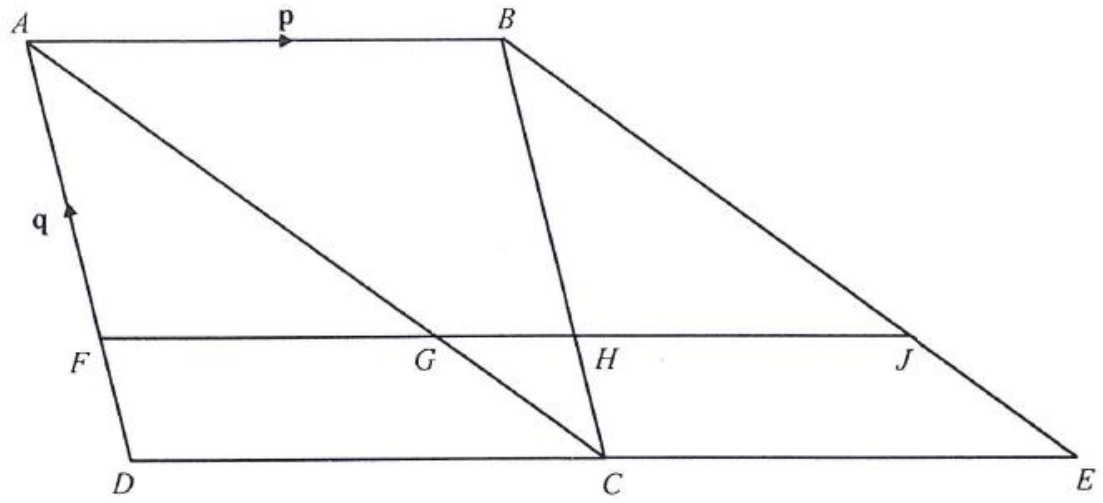
a)

$$\overrightarrow{RP} = \overrightarrow{RO} + \overrightarrow{OP} = \overrightarrow{QP} + \overrightarrow{OP} = \begin{pmatrix}-2\\-4\end{pmatrix} + \begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}1\\-2\end{pmatrix}$$

b)

$$\overrightarrow{OJ} = \overrightarrow{OP} + \overrightarrow{PJ} = \begin{pmatrix}3\\2\end{pmatrix} + m\begin{pmatrix}1\\-2\end{pmatrix} = \begin{pmatrix}3+m\\2-2m\end{pmatrix}$$

Question 18:



$ABCD$ is a rhombus. $ABEC$ is a parallelogram. FJ is parallel to AB and DC. F, G, H and J are points on AD, AC, BC and BE respectively.

$\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{DA} = \mathbf{q}$. $FG : DC = 8 : 11$.

a) Show that triangle ABC and JHB are similar. Give a reason for each statement you make.

b) Express $\overrightarrow{CG}$, as simply as possible, in terms of $\mathbf{p}$ and $\mathbf{q}$.

c) X is a point on BE. Triangle BCX is congruent to triangle ECX. Find $\overrightarrow{CX}$, as simply as possible in terms of $\mathbf{p}$ and $\mathbf{q}$.

d) Find the ratio $BJ : JE$

e) Find the ratio of area triangle BHJ : area quadrilateral $ABHG$.

a)

$$\angle ABC = \angle JHB \text{ (alt angle)}$$

$$\angle ACB = \angle JBH \text{ (alt angle)}$$

By AA similarity test, triangle ABC is similar to triangle JHB

b)

$$\overrightarrow{CA} = \overrightarrow{CB} + \overrightarrow{BA} = \underset{\sim}{q} - \underset{\sim}{p}$$

$$\overrightarrow{CG} = \frac{3}{11}\left(\underset{\sim}{q} - \underset{\sim}{p}\right) = \frac{3}{11}\underset{\sim}{q} - \frac{3}{11}\underset{\sim}{p}$$

c) X is the midpoint of BE

$$\overrightarrow{CX} = \overrightarrow{CE} + \overrightarrow{EX} = \underset{\sim}{p} + \frac{1}{2}\overrightarrow{CA} = \underset{\sim}{p} + \frac{1}{2}\left(\underset{\sim}{q} - \underset{\sim}{p}\right) = \frac{1}{2}\underset{\sim}{p} + \frac{1}{2}\underset{\sim}{q}$$

d)

$$BJ : JE$$

$$8 : 3$$

e)

$$\text{Area } BHJ : \text{Area } ABHG$$

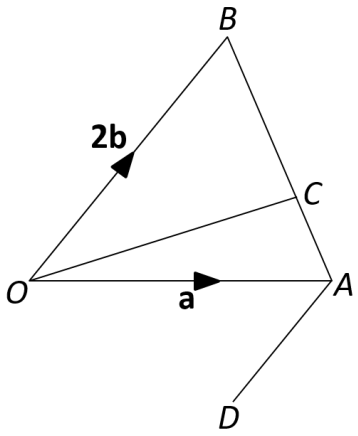
$$\frac{1}{2} \times HJ \times h : \frac{1}{2} \times (GH + AB) \times h$$

$$HJ : (GH + AB)$$

$$8 : 14$$

$$4 : 7$$

Question 19:



The position vectors of *A* and *B*, relative to *O*, are **a** and 2**b** respectively. $\overrightarrow{BC}=\frac{3}{4}\overrightarrow{BA}$ and $\overrightarrow{AD}=\frac{1}{2}\overrightarrow{BO}$.

Find, in terms of **a** and **b**,

a) $\overrightarrow{AB}$

b) $\overrightarrow{OC}$

c) the position vector of *P*, such that $\overrightarrow{BP}=3\overrightarrow{OA}$.

d) What can you deduce about *O*, *C* and *P*? Explain your answer.

Find the ratio of

e) the area of triangle *OCB* to the area of triangle *OCA*,

f) the area of triangle *OAC* to the area of triangle *OAD*.

a)

$$\overrightarrow{AB}=\overrightarrow{AO}+\overrightarrow{OB}=-\underset{\sim}{a}+2\underset{\sim}{b}$$

b)

$$\overrightarrow{OC}=\overrightarrow{OA}+\overrightarrow{AC}=\underset{\sim}{a}+\frac{1}{4}\left(-\underset{\sim}{a}+2\underset{\sim}{b}\right)=\frac{3}{4}\underset{\sim}{a}+\frac{1}{2}\underset{\sim}{b}$$

c)

$$\overrightarrow{OP} = \overrightarrow{OB} + \overrightarrow{BP} = \overrightarrow{OB} + 3\overrightarrow{OA} = 2\underset{\sim}{b} + 3\underset{\sim}{a}$$

d)

$$\overrightarrow{OC} = \frac{3}{4}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b} = \frac{1}{4}\left(2\underset{\sim}{b} + 3\underset{\sim}{a}\right) = \frac{1}{4}\overrightarrow{OP}$$

Since OC is parallel to OP and O is a common point, O, C and P are collinear.

e)

$$\text{Area of } OCB : \text{Area of } OCA$$

$$\frac{1}{2} \times CB \times h : \frac{1}{2} \times CA \times h$$

$$CB : CA$$

$$3 : 1$$

f)

$$\frac{\text{Area of } OAC}{\text{Area of } ODAB}$$

$$= \frac{\text{Area of } OAC}{\text{Area of } OAB} \times \frac{\text{Area of } OAB}{\text{Area of } ODAB}$$

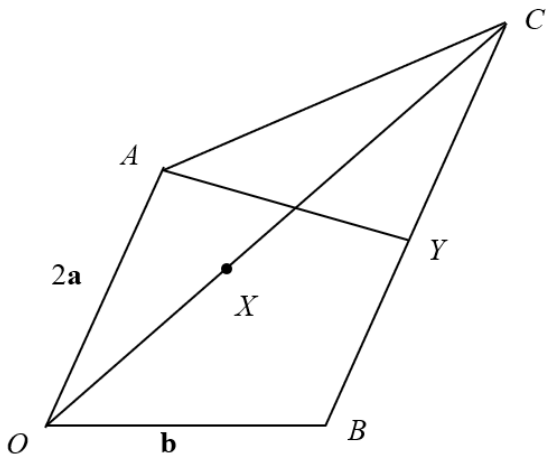
$$= \frac{1}{4} \times \frac{2}{3}$$

$$= \frac{1}{6}$$

$$\text{Area of } OAC : \text{Area of } OAD$$

$$1 : 2$$

Question 20:



In the diagram, $\overrightarrow{OA}$ = 2**a**, $\overrightarrow{OB}$ = **b**. *BC* is parallel to *OA* and *BC* = $\frac{3}{2}OA$. *X* is a point on *OC* such that *OX* = $\frac{2}{3}XC$. *Y* is the midpoint of *BC*. Express in terms of **a** and/or **b,** as simply as possible,

a) $\overrightarrow{AB}$

b) $\overrightarrow{OC}$

c) $\overrightarrow{OX}$

d) $\overrightarrow{AX}$

e) What can you deduce about the points *A*, *X* and *B* ? Justify your answer.

f) *AY* produced meets *OB* produced at a point *Z*. Given that $\overrightarrow{AZ} = h\overrightarrow{AY}$, express $\overrightarrow{AZ}$ in terms of **a**, **b** and *h*.

g) Given also that $\overrightarrow{OZ} = k\overrightarrow{OB}$, express $\overrightarrow{AZ}$ in terms of **a**, **b**,and *k*.

h) Hence, show that *h* = 4 and *k* = 4.

Find the value of

i) $\frac{\text{area of } \Delta OAX}{\text{area of } \Delta OAC}$

j) $\frac{\text{area of } \Delta OBX}{\text{area of } \Delta ABC}$

a)

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -2\underset{\sim}{a} + \underset{\sim}{b}$$

b)

$$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \underset{\sim}{b} + 3\underset{\sim}{a}$$

c)

$$\overrightarrow{OX} = \frac{2}{5}\overrightarrow{OC} = \frac{2}{5}\underset{\sim}{b} + \frac{6}{5}\underset{\sim}{a}$$

d)

$$\overrightarrow{AX} = \overrightarrow{AO} + \overrightarrow{OX} = -2\underset{\sim}{a} + \frac{2}{5}\underset{\sim}{b} + \frac{6}{5}\underset{\sim}{a} = -\frac{4}{5}\underset{\sim}{a} + \frac{2}{5}\underset{\sim}{b}$$

e)

$$\overrightarrow{AX} = -\frac{4}{5}\underset{\sim}{a} + \frac{2}{5}\underset{\sim}{b} = \frac{2}{5}\left(-2\underset{\sim}{a} + \underset{\sim}{b}\right) = \frac{2}{5}\overrightarrow{AB}$$

AX is parallel to AB and A is a common point. So, A, X and B are collinear.

f)

$$\overrightarrow{AZ} = h\left(\overrightarrow{AB} + \overrightarrow{BY}\right) = h\left(-2\underset{\sim}{a} + \underset{\sim}{b} + \frac{3}{2}\underset{\sim}{a}\right) = h\left(\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}\right)$$

g)

$$\overrightarrow{AZ} = \overrightarrow{AO} + \overrightarrow{OZ} = -2\underset{\sim}{a} + k\underset{\sim}{b}$$

h)

$$-2\underset{\sim}{a} + k\underset{\sim}{b} = h\left(\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}\right)$$

$$-2 = -\frac{1}{2}h$$

$$h = 4$$

$$k = 4$$

i)

$$\frac{\text{Area of } OAX}{\text{Area of } OAC} = \frac{\frac{1}{2} \times OX \times h}{\frac{1}{2} \times OC \times h} = \frac{OX}{OC} = \frac{2}{5}$$

j)

$$\frac{\text{Area of } OBX}{\text{Area of } ABC}$$

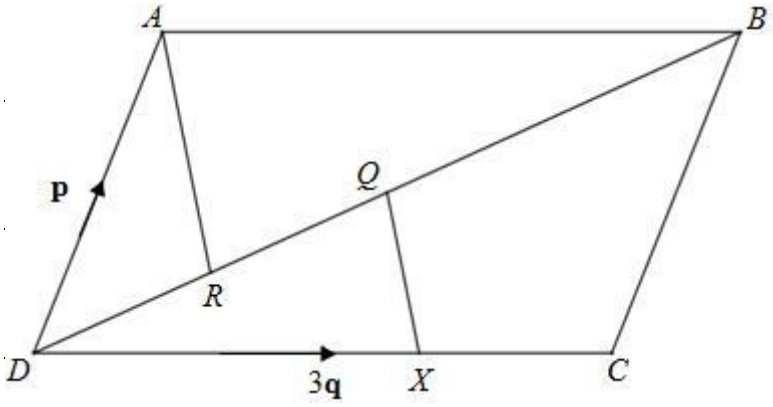
$$= \frac{\text{Area of } OBX}{\text{Area of } BXC} \times \frac{\text{Area of } BXC}{\text{Area of } ABC}$$

$$= \frac{2}{3} \times \frac{3}{5}$$

$$= \frac{2}{5}$$

Question 21:

In the diagram, *ABCD* is a parallelogram, *Q* is the midpoint of *DB*, *R* is the midpoint of *DQ* and *X* is the point on *DC* such that *DX* = 2*XC*.



a) Given that $\overrightarrow{DA} = \mathbf{p}$ and $\overrightarrow{DC} = 3\mathbf{q}$, show that $\overrightarrow{DB} = \mathbf{p} + 3\mathbf{q}$.

b) $\overrightarrow{DR}$

c) $\overrightarrow{AR}$,

d) $\overrightarrow{QX}$.

e) Write down two facts about your answers to **(b)** and **(c)**.

f) What is the special name given to the quadrilateral *AQXR*?

Write down the value of

g) $\dfrac{\text{area of } \Delta ABR}{\text{area of } \Delta DQX}$

h) $\dfrac{\text{area of } \Delta DAR}{\text{area of parallelogram } ABCD}$

a)

$$\overrightarrow{DB} = \overrightarrow{DA} + \overrightarrow{AB} = \underset{\sim}{p} + 3\underset{\sim}{q}$$

b)

$$\overrightarrow{DR} = \frac{1}{4}\overrightarrow{DB} = \frac{1}{4}\underset{\sim}{p} + \frac{3}{4}\underset{\sim}{q}$$

c)

$$\overrightarrow{AR} = \overrightarrow{AD} + \overrightarrow{DR} = -\underset{\sim}{p} + \frac{1}{4}\underset{\sim}{p} + \frac{3}{4}\underset{\sim}{q} = -\frac{3}{4}\underset{\sim}{p} + \frac{3}{4}\underset{\sim}{q}$$

d)

$$\overrightarrow{QX} = \overrightarrow{QD} + \overrightarrow{DX} = -\frac{1}{2}\underset{\sim}{p} - \frac{3}{2}\underset{\sim}{q} + 2\underset{\sim}{q} = -\frac{1}{2}\underset{\sim}{p} + \frac{1}{2}\underset{\sim}{q}$$

e)

$$\overrightarrow{AR} = -\frac{3}{4}\underset{\sim}{p} + \frac{3}{4}\underset{\sim}{q} = \frac{3}{2}\left(-\frac{1}{2}\underset{\sim}{p} + \frac{1}{2}\underset{\sim}{q}\right) = \frac{3}{2}\overrightarrow{QX}$$

AR is parallel to QX. AR is 1.5 times longer than QX.

f) Trapezium

g)

$$\frac{\text{Area of } ABR}{\text{Area of } DQX} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

h)

$$\frac{\text{Area of } DAR}{\text{Area of } DAB} = \frac{1}{4}$$

$$\frac{\text{Area of } DAR}{\text{Area of } ABCD} = \frac{1}{8}$$