EQUITY
LEARNING PLACE

Sec 4 Differentiation Revision

1) Find the equation of normal to the curve $y = \sqrt{2-x}$ at $x = 1$. [3]

$y = \sqrt{2-x}$

$\frac{dy}{dx} = \frac{1}{2}(2-x)^{-\frac{1}{2}}(-1)$

$= -\frac{1}{2\sqrt{2-x}}$

when $x = 1$

$\frac{dy}{dx} = -\frac{1}{2}$, $y = 1$

Gradient of normal $= 2$

$1 = 2(1) + c$

$c = -1$

Eqn of normal

$y = 2x - 1$

2) Given that $f(x) = \frac{3x}{x^2+1}$, find the range of values of x for which $f(x)$ is increasing. [5]

$f'(x) = \frac{3(x^2+1) - 3x(2x)}{(x^2+1)^2}$

$= \frac{-3x^2+3}{(x^2+1)^2}$

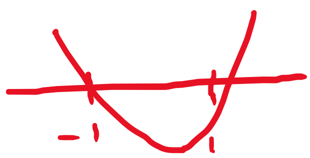
For $f(x)$ to be increasing, $f'(x) > 0$

$\frac{-3x^2+3}{(x^2+1)^2} > 0$

$-3x^2 + 3 > 0$

$3x^2 - 3 < 0$

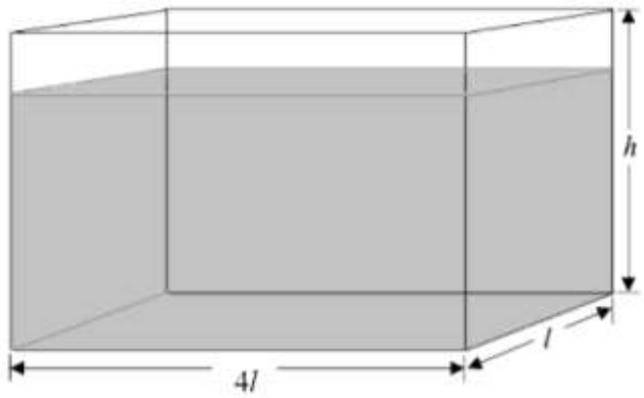
$3(x+1)(x-1) < 0$



$\therefore -1 < x < 1$

Sec 4 Differentiation Revision

3) Jevier constructed an opened fish tank with a rectangular base of length $4l$ m, breadth l m and height h m. He wanted the total outer surface area of the fish tank to be 5 m^2.



i) Show that the volume of the tank, V m^3, is given by $V = \frac{2}{5}(5l - 4l^3)$. [3]

$V = 4l \times l \times h$

$= 4l^2 h \quad -(1)$

$A = 4l \times l + 2(4l)(h) + 2(l)(h)$

$5 = 4l^2 + 10lh$

$5 - 4l^2 = 10lh$

$\frac{1}{2l} - \frac{2l}{5} = h \quad -(2)$

Sub (2) into (1),

$V = 4l^2\left(\frac{1}{2l} - \frac{2l}{5}\right)$

$= 2l - \frac{8}{5}l^3 \quad = \frac{2}{5}(5l - 4l^3)$

ii) Determine the area of the rectangular base for the tank to contain the maximum amount of water when filled to the brim. [4]

$V = 2l - \frac{8}{5}l^3$

$\frac{dV}{dl} = 2 - \frac{24}{5}l^2$

when $\frac{dV}{dl} = 0$,

$2 - \frac{24}{5}l^2 = 0$

$\frac{24}{5}l^2 = 2$

$l^2 = \frac{5}{12}$

$l = \sqrt{\frac{5}{12}}$

$\frac{d^2V}{dl^2} = -\frac{48}{5}l$

Since $l > 0$,

$\frac{d^2V}{dl^2} < 0$

$l = \sqrt{\frac{5}{12}}$ will give max. volume

Area of rectangular base $= 4l^2 = 4 \times \frac{5}{12}$

$= \frac{5}{3} m^2$

Sec 4 Differentiation Revision

4a) Show that $y = \ln\left(\frac{8+3x}{3x-4}\right)$ has no turning point for all values of x. [4]

$y = \ln(8+3x) - \ln(3x-4)$

$\frac{dy}{dx} = \frac{3}{8+3x} - \frac{3}{3x-4}$

$= \frac{9x-12 - 24-9x}{(8+3x)(3x-4)}$

$= \frac{-36}{(8+3x)(3x-4)}$

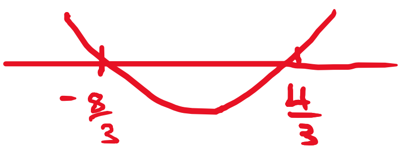
Since $\frac{dy}{dx} \neq 0$ for all values of x, there is no turning pt.

b) Find the range of values of x in which the graph $y = \ln\left(\frac{8+3x}{3x-4}\right)$ is decreasing. [3]

For y to be decreasing, $\frac{dy}{dx} < 0$

$\frac{-36}{(8+3x)(3x-4)} \leq 0$

$(8+3x)(3x-4) > 0$



$\therefore x < -\frac{8}{3}$ or $x > \frac{4}{3}$

5) The equation of a curve is $y = \frac{2\sin x}{\cos x - 3}$

a) Express $\frac{dy}{dx}$ in the form $\frac{a + b\cos x}{(\cos x - 3)^2}$, where a and b are constants. [3]

$$\frac{dy}{dx} = \frac{(2\cos x)(\cos x - 3) - 2\sin x(-\sin x)}{(\cos x - 3)^2}$$

$$= \frac{2\cos^2 x - 6\cos x + 2\sin^2 x}{(\cos x - 3)^2}$$

$$= \frac{2 - 6\cos x}{(\cos x - 3)^2}$$

b) Given y is decreasing at 0.064 units per second, find the rate of change of x with respect to time when $x = \frac{\pi}{3}$. [3]

$$\frac{dx}{dt} = \frac{dx}{dy} \times \frac{dy}{dt}$$

$$= \frac{(\cos x - 3)^2}{2 - 6\cos x} \times (-0.064)$$

when $x = \frac{\pi}{3}$,

$$= \frac{(\frac{1}{2} - 3)^2}{2 - 6(\frac{1}{2})} \times (-0.064)$$

$$= 0.4$$

$\therefore$ rate of change of x is 0.4 units per second

6) Find the derivative of $y = \frac{e^{\frac{1}{x}}}{2x+3}$, and determine whether y is an increasing or decreasing function. [7]

$$\frac{dy}{dx} = \frac{-\frac{1}{x^2}e^{\frac{1}{x}}(2x+3) - e^{\frac{1}{x}}(2)}{(2x+3)^2}$$

$$= \frac{-\frac{1}{x^2}e^{\frac{1}{x}}\left[(2x+3) + 2x^2\right]}{(2x+3)^2}$$

$$= \frac{-e^{\frac{1}{x}}(2x^2+2x+3)}{x^2(2x+3)^2}$$

$$= \frac{-2e^{\frac{1}{x}}\left[x^2+x+\frac{3}{2}\right]}{x^2(2x+3)^2}$$

$$= \frac{-2e^{\frac{1}{x}}\left[(x+\frac{1}{2})^2-(\frac{1}{2})^2+\frac{3}{2}\right]}{x^2(2x+3)^2}$$

$$= \frac{-2e^{\frac{1}{x}}\left[(x+\frac{1}{2})^2+\frac{5}{4}\right]}{x^2(2x+3)^2}$$

Since $x^2 > 0$, $(2x+3)^2 > 0$, $(x+\frac{1}{2})^2+\frac{5}{4} > 0$

and $e^{\frac{1}{x}} > 0$, $\frac{dy}{dx} < 0$ for all values of x

Hence, y is a decreasing function.