EQUITY
LEARNING PLACE

NAME : ______________________( )   CLASS : __________________

MARKS : _________ / 30

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1) The path of a water jet can be modelled by the quadratic function

$$y = C(x - 1.2)^2 + 2.25$$

where x m is the horizontal distance it travels, y m is the height of the water above the ground and C is a constant. The initial height of the water jet is 1.05 m above the ground.

i) Find the value of C. [2]

when $x=0$, $y=1.05$

$1.05 = C(-1.2)^2 + 2.25$

$-1.2 = 1.44C$

$C = -\frac{5}{6}$

ii) Find the maximum height above the ground that the water jet reaches. [1]

max height $= 2.25$ m

iii) Find the value of x for which the water jet is 1.05 m above the ground again. [2]

when $y=1.05$

$1.05 = -\frac{5}{6}(x-1.2)^2 + 2.25$

$(x-1.2)^2 = \frac{1.05 - 2.25}{-\frac{5}{6}}$

$(x-1.2)^2 = 1.44$

$x - 1.2 = 1.2$ or -1.2

$x = 2.4$ or 0

$\therefore x = 2.4$

iv) Find the maximum horizontal distance travelled by the water jet. [2]

$y = -\frac{5}{6}(x-1.2)^2 + 2.25$

when $y=0$,

$(x-1.2)^2 = \frac{2.25}{\frac{5}{6}}$

$x - 1.2 = \pm\sqrt{2.7}$

$x = 2.84$ or -4.00

$\therefore$ max dist $= 2.84$m

2) Prove that $2x^2 - 5x + 24 > 0$ for all real values of x. Hence, find the range of values of x for which $\dfrac{3x^2 - 16x + 5}{2x^2 - 5x + 24} < 0$. [7]

$2x^2 - 5x + 24$

$= 2(x^2 - \frac{5}{2}x + 12)$

$= 2[(x - \frac{5}{4})^2 - \frac{25}{16} + 12]$

$= 2[(x - \frac{5}{4})^2 + \frac{167}{16}]$

$= 2(x - \frac{5}{4})^2 + \frac{167}{8}$

Since $(x - \frac{5}{4})^2 \geq 0$,

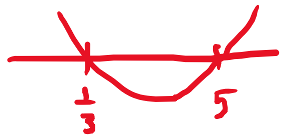
$2(x - \frac{5}{4})^2 + \frac{167}{8} \geq \frac{167}{8} > 0$

$\dfrac{3x^2 - 16x + 5}{2x^2 - 5x + 24} < 0$

Since $2x^2 - 5x + 24 > 0$ for all x,

$3x^2 - 16x + 5 < 0$

$(3x - 1)(x - 5) < 0$



$\therefore \frac{1}{3} < x < 5$

3) Express $\dfrac{2x^3+6x^2+1}{(x-1)(x+2)^2}$ in partial fractions. [5]

$(x-1)(x+2)^2$

$= (x-1)(x^2+4x+4)$

$= x^3+3x^2-4$

$$\begin{array}{r|l} & \quad 2 \\ \hline x^3+3x^2-4 & 2x^3+6x^2+1 \\ & 2x^3+6x^2-8 \\ \hline & \qquad\quad 9 \end{array}$$

$$\therefore \frac{2x^3+6x^2+1}{(x-1)(x+2)^2} = 2 + \frac{9}{(x-1)(x+2)^2}$$

$$\frac{9}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$9 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$$

when $x=1$,	when $x=-2$	when $x=0$,
$9 = 9A$	$9 = -3C$	$9 = 4 - 2B + 3$
$A = 1$	$C = -3$	$-2B = 2$
		$B = -1$

$$\therefore \frac{2x^3+6x^2+1}{(x-1)(x+2)^2} = 2 + \frac{1}{x-1} - \frac{1}{x+2} - \frac{3}{(x+2)^2}$$

4) The polynomial $f(x) = 3x^3 + ax^2 + bx + 1$, where a and b are constants, is such that it is divisible by $x+1$ and leaves a remainder of 27 when divided by $x-2$.

a) Show that $a = 1$ and $b = -1$. [4]

$f(x) = 3x^3 + ax^2 + bx + 1$

$f(-1) = -3 + a - b + 1$

$= a - b - 2$

$a - b - 2 = 0$

$a = b + 2$ — (1)

$f(2) = 24 + 4a + 2b + 1$

$4a + 2b + 25 = 27$

$4a + 2b = 2$ — (2)

Sub (1) into (2)

$4b + 8 + 2b = 2$

$b = -1$

Sub $b = -1$ into (1)

$a = -1 + 2$

$= 1$

b) Explain why the equation $f(x) = 0$ has only 1 real root and state its value. [4]

$f(x) = 3x^3 + x^2 - x + 1$

when $f(x) = 0$,

$3x^3 + x^2 - x + 1 = 0$

$$
\begin{array}{r|l}
 & 3x^2 - 2x + 1 \\
x+1 & 3x^3 + x^2 - x + 1 \\
 & 3x^3 + 3x^2 \\
\hline
 & -2x^2 - x + 1 \\
 & -2x^2 - 2x \\
\hline
 & x + 1 \\
 & x + 1 \\
\hline
 & 0
\end{array}
$$

$(x+1)(3x^2 - 2x + 1) = 0$

$x + 1 = 0$ or $3x^2 - 2x + 1 = 0$

$x = -1$

$(-2)^2 - 4(3)(1)$

$= -8$

Since $b^2 - 4ac < 0$, there is no solution

Hence $x = -1$ is the only solution.

5) Factorise $3x^3 - 24y^3$ completely. [3]

$3x^3 - 24y^3$

$= 3(x^3 - 8y^3)$

$= 3(x-2y)(x^2 + x(2y) + 4y^2)$

$= 3(x-2y)(x^2 + 2xy + 4y^2)$

End of paper