

EQUITY

LEARNING PLACE

Elementary Math Topical (Number Pattern)

Question 1:

Consider the sequence 5, 7, 9, 11, 13, ...

- a) Write down the sixth term in the sequence.
- b) Write down an expression, in terms of n , for the n th term of the sequence.

a) 15

b) $2n + 3$

Question 2:

The first four terms of a sequence are as follows:

78 75 72 69

- a) Write down the tenth term of the sequence.
- b) Write down an expression, in terms of n , for the n th term of the sequence.

a) 51

b) $-3n + 81$

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Question 3:

The first four terms in a sequence are 45, 41, 37 and 33.

- a) Find an expression, in terms of n , for the n th term, of this sequence.
- b) Evaluate the 25th term in the sequence.
- c) Determine whether the number, -11 , is a term of this sequence

a) $-4n + 49$

b) $-4(25) + 49 = -51$

c)

$$-4n + 49 = -11$$

$$-4n = -60$$

$$n = 15$$

Yes, -11 is the 15th term of the sequence.

Question 4:

These are the first four terms in a sequence.

3, 6, 11, 18, ...

- a) Write down the eighth term in the sequence.
- b) Write down an expression, in terms of n , for the n th term in the sequence.

a) 66

b) $n^2 + 2$

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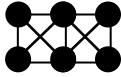
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Question 5:

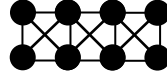
The following diagrams show the number of sticks and dots required to form a series of patterns.



Step 1



Step 2



Step 3

i) Complete the table below:

Step Number (n)	Number of stick (S)	Number of dots (D)
1	6	4
2	11	6
3	16	8
4	21	10
5	26	12

ii) Without drawing further, determine the number of sticks in Step 10.

iii) Without drawing further, determine the value of n that will result in 48 dots.

iv) Derive a formula for finding the number of sticks (S) for step n .

ii) 51

iii) 22

iv) $5n + 1$

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Question 6:

$$3^2 + 4^2 = x^2$$

$$3^3 + 4^3 + 5^3 = y^3$$

- a) Find the values of x and y in the above equations.
- b) Alan commented that he is able to write down the next line following the above pattern. Write down the next line of this pattern.
- c) By considering powers of even and odd numbers, explain, without calculating the exact value of each term, why Alan's conjecture in **(b)** is not true.

a)

$$3^2 + 4^2 = 25 = 5^2$$

$$x = 5$$

$$3^3 + 4^3 + 5^3 = 216 = 6^3$$

$$y = 6$$

b)

$$3^4 + 4^4 + 5^4 + 6^4 = 7^4$$

c)

3^4 is an odd number

4^4 is an even number

5^4 is an odd number

6^4 is an even number

$3^4 + 4^4 + 5^4 + 6^4$ is an even number

7^4 is an odd number

\therefore Alan's conjecture is not true

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Question 7:

Consider the following pattern.

$$\begin{aligned}1 - \frac{1}{2} &= \frac{1}{2} \\ \frac{1}{2} - \frac{1}{3} &= \frac{1}{6} \\ \frac{1}{3} - \frac{1}{4} &= \frac{1}{12} \\ \frac{1}{4} - \frac{1}{5} &= a\end{aligned}$$

- a) State the value of a .
- b) Write down the 10th line of the pattern.
- c) Write down the n^{th} line of the pattern.
- d) Use the pattern to find the sum of $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \dots + \frac{1}{9900}$.

a) 20

b)

$$\frac{1}{10} - \frac{1}{11} = \frac{1}{121}$$

c)

$$\frac{1}{n} - \frac{1}{n+1} = \frac{1}{n(n+1)}$$

d)

$$\begin{aligned}& \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \dots + \frac{1}{9900} \\ &= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \dots + \left(\frac{1}{99} - \frac{1}{100}\right) \\ &= 1 + \left(-\frac{1}{2} + \frac{1}{2}\right) + \left(-\frac{1}{3} + \frac{1}{3}\right) + \left(-\frac{1}{4} + \frac{1}{4}\right) + \dots + \left(-\frac{1}{99} + \frac{1}{99}\right) - \frac{1}{100} \\ &= 1 - \frac{1}{100} = \frac{99}{100}\end{aligned}$$

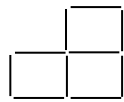
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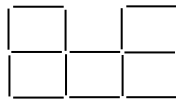
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Question 8:

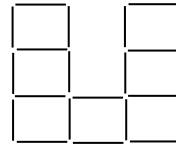
Sticks of equal length are used to form the patterns as shown.



Figure



Figure



Figure

a) Draw Figure 5. (Draw Yourself 😊)

b) The table below shows the relationship between the figure, the number of squares formed and the number of sticks used. Complete the table for $n = 5$ and $n = 6$.

Figure (n)	No. of squares formed (Q)	No. of sticks used (T)
1	3	10
2	5	16
3	7	22
4	9	28
5	11	34
6	13	40

c) Express T in terms of n .

d) Hence, find the number of sticks used for Figure 30.

c) $T = 6n + 4$

d) $T = 6(30) + 4 = 184$

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Question 9:

The first three terms in a sequence of numbers, T_1, T_2, T_3, \dots are given below.

$$T_1 = 1^2 + 7 = 8$$

$$T_2 = 2^2 + 14 = 18$$

$$T_3 = 3^2 + 21 = 30$$

- a) Find T_4 .
- b) Find an expression, in terms of n , for T_n .
- c) Evaluate T_{50} .

The first four terms in a different sequence of numbers, P_1, P_2, P_3, \dots are

$$8, 13, 18, 23, \dots$$

- d) Find an expression, in terms of n , for the n th term, P_n , of this sequence.
- e) Explain why the number 102 is not in this sequence.
- f) By forming an equation in terms of n , find the value(s) of n such that $\frac{T_n}{P_n} = 1$.

$$a) T_4 = 4^2 + 28 = 44$$

$$b) T_n = n^2 + 7n$$

$$c) T_{50} = 50^2 + 7(50) = 2850$$

$$d) P_n = 5n + 3$$

e)

$$5n + 3 = 102$$

$$5n = 99$$

$$n = \frac{99}{5}$$

Since n is not a positive integer, 102 is not a term in the sequence.

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f)

$$\frac{(n^2 + 7n)}{5n + 3} = 1$$

$$n^2 + 7n = 5n + 3$$

$$n^2 + 2n - 3 = 0$$

$$(n + 3)(n - 1) = 0$$

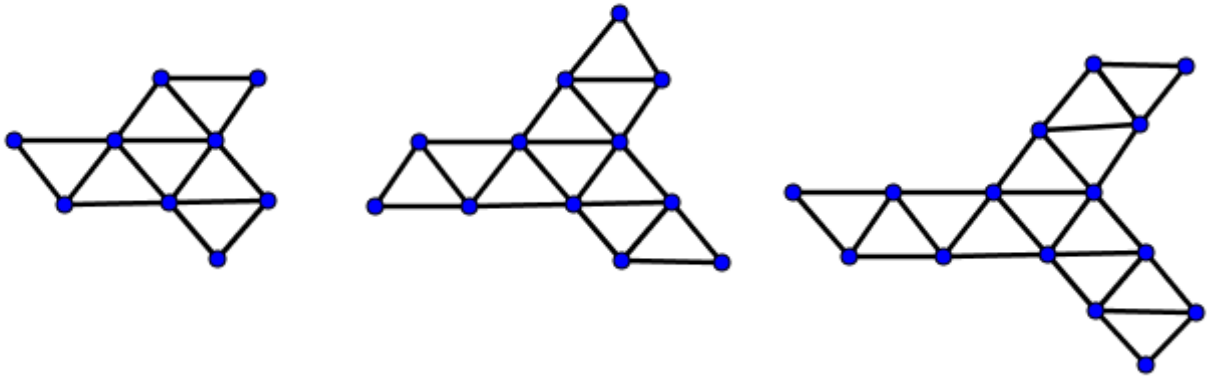
$$n = -3 \text{ or } n = 1$$

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Question 10:



1st pattern

2nd pattern

3rd pattern

Pattern	Number of triangles, T	Number of straws, S	Number of thumbtacks, t
1 st	7	15	9
2 nd	10	21	12
3 rd	13	27	15
4 th	16	33	18
5 th	x	y	z
n^{th}

- (a) Write down the value of x , of y and of z .
- (b) In the n^{th} pattern, express in terms of n ,
 - (i) the number of triangles formed,
 - (ii) the number of straws used.
- (c) In which pattern would 108 thumbtacks be used?
- (d) Explain why the value of 605 could not appear in Column S of the table.
- (e) Write down an expression connecting
 - (i) S and T ,
 - (ii) S , T and t .

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a) $x = 19, y = 39, z = 21$

bi) $3n + 4$

bii) $6n + 9$

c) $t = 3n + 6$

$$3n + 6 = 108$$

$$3n = 102$$

$$n = 34$$

d)

$$3n + 4 = 605$$

$$3n = 601$$

$$n = 200\frac{1}{3}$$

Since n is not a positive integer, 605 is not a term in column S .

ei) $T = 3n + 4 \rightarrow 3n = T - 4$

$$S = 6n + 9$$

$$S = 2(T - 4) + 9$$

$$S = 2T + 1$$

eii) $t = 3n + 6$

$$t = T + 2$$

$$2S = 4T + 2$$

$$2S = 4T + t - T$$

$$2S = 3T - t$$

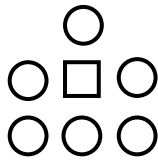
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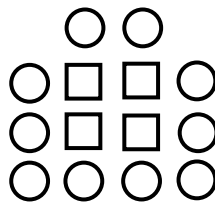
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Question 11:

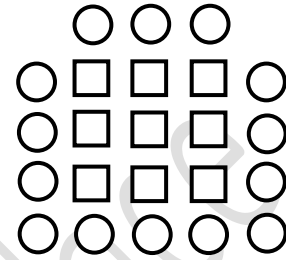
Alice arranges tables and chairs according to the number of people attending the meeting in the following arrangement.



Layout 1



Layout 2



Layout 3

The number of tables and chairs are recorded in the table below.

Layout Number (n)	Number of tables (t)	Number of chairs (c)
1	1	6
2	4	10
3	9	14
\vdots	\vdots	\vdots
7	a	b

- (a) Write down the values of a and b .
- (b) Write down an equation connecting n and t .
- (c) Write down an equation connecting n and c .
- (d) Is it possible for the value of c to be 50? Explain your answer.

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a) $a = 49, b = 30$

b) $t = n^2$

c) $c = 4n + 2$

d)

$$4n + 2 = 50$$

$$4n = 48$$

$$n = 12$$

Yes, 50 is the 12th term of c

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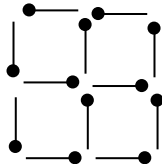
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Question 12:

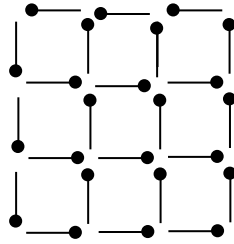
John used match sticks to make a series of squares. The first three squares he constructed are as shown below.



$n = 1$



$n = 2$



$n = 3$

For each square, T represents the number of match sticks used, S the total number of small squares formed and P the number of points at which 2 or more match sticks meet. The values of T , S and P are tabulated as shown in the following.

n	Number of match sticks used T_n	Number of small squares formed S_n	Number of points at which 2 or more match sticks meet P_n
1	4	1	4
2	12	4	9
3	24	9	16
4	x	y	z

- (a) Write down the values of x , y and z .
- (b) Find an expression, in terms of n , for S_n and P_n .
- (c) Find the value of P when $S = 36$.
- (d) Write down the formula connecting T_n , S_n and P_n .
- (e) Hence or otherwise, find the value of P when $T = 364$ and $S = 169$.
- (f) Give a reason why the number 112 cannot appear in column S .

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a) $x = 40, y = 16, z = 25$

b) $S_n = n^2, P_n = (n + 1)^2$

c)

$$S = 36$$

$$n = 6$$

$$P = 7^2 = 49$$

d)

$$T_n = S_n + P_n - 1$$

e)

$$364 = 169 + P_n - 1$$

$$P_n = 196$$

f) 112 is not a perfect square.

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Question 13:

The first four terms in a sequence of numbers, T_1 , T_2 , T_3 and T_4 are given below.

$$T_1 = 2(1)^2 + 1 = 3$$

$$T_2 = 2(2)^2 + 3 = 11$$

$$T_3 = 2(3)^2 + 5 = 23$$

$$T_4 = 2(4)^2 + 7 = 39$$

- (a) Write down an expression for T_5 and evaluate it.
- (b) Evaluate T_{25} .
- (c) Find an expression, in terms of n , for T_n .
- (d) What term will the number 2811 be in the sequence?
- (e) Explain why the number in the sequence cannot be an even number.

a) $T_5 = 2(5)^2 + 9 = 59$

b) $T_{25} = 2(25)^2 + 49 = 1299$

c) $T_n = 2n^2 + 2n - 1$

d)

$$2n^2 + 2n - 1 = 2811$$

$$2n^2 + 2n - 2812 = 0$$

$$n^2 + n - 1406 = 0$$

$$(n + 38)(n - 37) = 0$$

$$n = -38 \text{ or } n = 37$$

2811 is the 37th term

e) $2n^2$ is even, $2n$ is even. When 1 is subtracted from even number it will become odd.

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Question 14:

The first four terms in a sequence of numbers are given below.

$$T_1 = 4 - 3 = 1$$

$$T_2 = 9 - 6 = 3$$

$$T_3 = 16 - 9 = 7$$

$$T_4 = 25 - 12 = 13$$

a) Find T_5 .

b) Show that the n th term of the sequence T_n , is given by $T_n = n^2 - n + 1$.

c) T_p and T_{p+1} are consecutive terms in the sequence. Find and simplify an expression, in terms of p , for $T_{p+1} - T_p$.

a) $T_5 = 36 - 15 = 21$

b)

$$T_n = (n + 1)^2 - (3n)$$

$$T_n = n^2 + 2n + 1 - 3n$$

$$T_n = n^2 - n + 1$$

c)

$$T_p = p^2 - p + 1$$

$$T_{p+1} = (p + 1)^2 - (p + 1) + 1$$

$$T_{p+1} = p^2 + 2p + 1 - p - 1 + 1$$

$$T_{p+1} = p^2 + p + 1$$

$$T_{p+1} - T_p = p^2 + p + 1 - (p^2 - p + 1) = 2p$$

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Question 15:

Consider the number pattern.

$$\begin{aligned}1 &= 1^2 \\1 + 3 &= 2^2 \\1 + 3 + 5 &= 3^2 \\1 + 3 + 5 + 7 &= 4^2 \\&\vdots \\&\vdots\end{aligned}$$

a) Write down the 5th and 6th lines of the pattern.

Given that the n^{th} line is $1 + 3 + 5 + \dots + (2n - 1) = 484$,

b) find the number of terms which are added in the n^{th} line,

c) find the value of the last term added in the n^{th} line.

a)

$$1 + 3 + 5 + 7 + 9 = 5^2$$

$$1 + 3 + 5 + 7 + 9 + 11 = 6^2$$

b)

$$\text{Number of terms} = \sqrt{484} = 22$$

c)

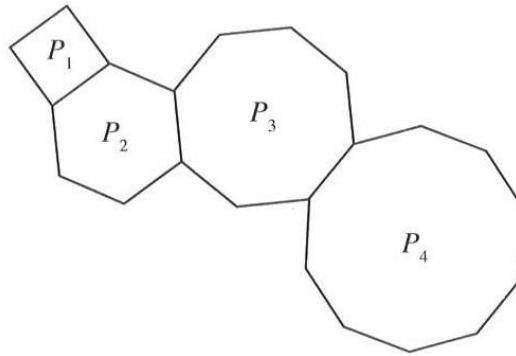
$$1 + 21(2) = 43$$

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Question 16:



The diagram above shows the first four polygons P_1 , P_2 , P_3 and P_4 in a sequence of connected polygons. The table below shows the number of sides each polygon has.

Polygon	P_1	P_2	P_3	P_4
No. of sides	4	6	8	10

- (i) How many sides does polygon P_6 have?
 - (ii) Find the size of one interior angle of polygon P_7 .
 - (iii) Write down an expression, in terms of n , for the number of sides P_n has.
- (b) The n th term of a sequence is given by $T_n = \frac{n(n+1)}{2}$.
- (i) Find the first three terms of the sequence.
 - (ii) Which term of the sequence has a value 210?
 - (iii) Explain why 100 cannot be a term of the sequence.

ai) 14

aii) $\frac{(7-2) \times 180}{7} = 128.6^\circ$

aiii) $P_n = 2n + 2$

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bi) 1, 3, 6

bii)

$$\frac{n(n+1)}{2} = 210$$

$$n^2 + n - 420 = 0$$

$$(n+21)(n-20) = 0$$

$$n = -21 \text{ or } n = 20$$

biii)

$$\frac{n(n+1)}{2} = 100$$

$$n^2 + n - 200 = 0$$

$$\left(n + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 200 = 0$$

$$n + \frac{1}{2} = \pm \sqrt{\frac{801}{4}}$$

$$n = -\frac{1}{2} \pm \sqrt{\frac{801}{4}}$$

$$n = 13.7 \text{ or } n = -14.7$$

Since n is not a positive integer, 100 is not a term in the sequence.

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Question 17:

The first four terms in a sequence of numbers, $a_1, a_2, a_3, a_4, \dots$, are given below.

$$a_1 = 3 - 3^0 = 2$$

$$a_2 = 5 - 3^1 = 2$$

$$a_3 = 7 - 3^2 = -2$$

$$a_4 = 9 - 3^3 = -18$$

a) Find an expression, in terms of n , for the n th term, a_n , of the sequence.

b) Explain why the value a_n will always be divisible by 2.

a)

$$a_n = 2n + 1 - 3^{n-1}$$

b)

-3^{n-1} is always odd number

$1 - 3^{n-1}$ will be an even number

$2n$ is an even number

$2n + 1 - 3^{n-1}$ will be an even number hence divisible by 2

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Question 18:

Consider the pattern

$$2^2 - 1^2 = 3$$

$$3^2 - 2^2 = 5$$

$$4^2 - 3^2 = 7$$

$$5^2 - 4^2 = 9$$

⋮

$$x^2 - y^2 = 321$$

⋮

a) Write down the seventh line in the pattern.

b) Find the value of $150^2 - 149^2$.

c) Find integer values of x and y which satisfy the equation $x^2 - y^2 = 321$.

a)

$$8^2 - 7^2 = 15$$

b)

$$150^2 - 149^2 = 299$$

c)

$$x^2 - y^2 = 321$$

$$161^2 - 160^2 = 321$$

$$x = 161, \quad y = 160$$