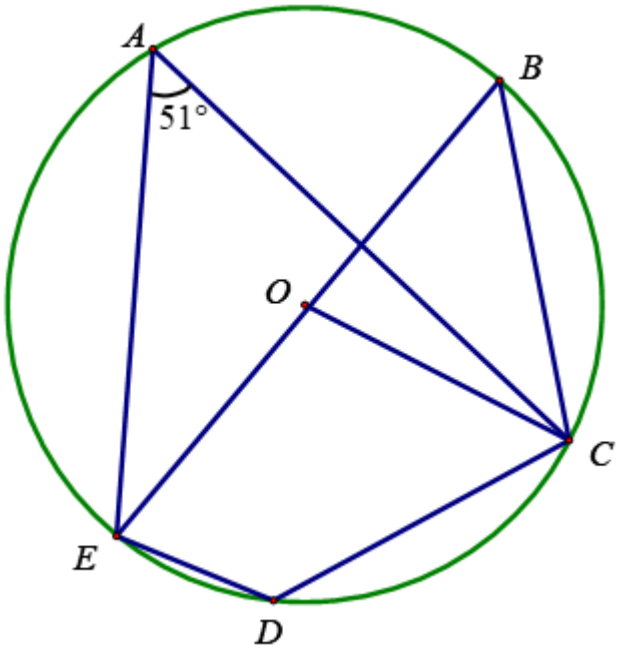
Elementary Math Topical (Properties of Circle)

Question 1:

In the diagram below, A, B, C, D and E are points on a circle with centre O.



Given that $\angle EAB = 51°$, find

a) $\angle EBC$

b) $\angle EOC$

c) $\angle EDC$.

a)

$\angle EBC = 51°$ (angle in same segment)

b)

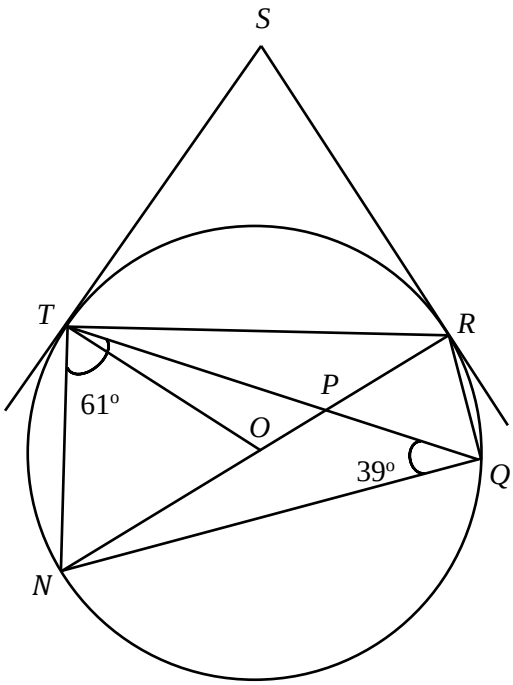
$\angle EOC = 102°$ (angle at centre = 2 angle at circumference)

c)

$\angle EDC = 180 - 51 = 129°$ (angle in opposite segment)

Equity Learning Place

Question 2:



In the above diagram, *TS* and *RS* are straight lines touching the circle, with centre *O*, at points *T* and *R* respectively. Points *T*, *R*, *Q* and *N* are points on the circumference of the circle. *NR* is the diameter of the circle. The straight line *TQ* intersects the diameter of the circle at point *P*. Angle *NTP* = 61° and angle *NQT* = 39°.

a) Give a reason to explain why angle *NQR* is a right angle. Hence, find angle *TNR*, stating your reasons clearly.

Calculate the following angles, stating your reasons clearly.

b) Angle *QTR*,

c) Angle *TON*,

d) Angle *TSR*.

e) It is given that the length of the straight line *TN* = 10 cm, find the area of the circle with centre at *O*.

Equity Learning Place

a) $\angle NQR$ is a right-angled triangle because of the property, right angle in semi-circle.

$$\angle PQR = 90 - 39 = 51°$$

$$\angle TNR = 51° \text{ (angle in same segment)}$$

b)

$$\angle QTR = 90 - 61 = 29° \text{ (right angle in semi circle)}$$

c)

$$\angle TON = 2 \times 39 = 78° \text{ (angle at centre} = 2 \text{ angle at circumference)}$$

d)

$$\angle TOQ = 180 - 78 = 102° \text{ (angle on a st. line)}$$

$$\angle TSR = 360 - 90 - 90 - 102 = 78° \text{ (sum of angle in quadrilateral)}$$

e)

$$\sin 39 = \frac{10}{NR}$$

$$NR = 15.89015$$

$$\text{Area} = \pi\left(\frac{15.89015}{2}\right)^2 = 198 \text{ cm}^2$$

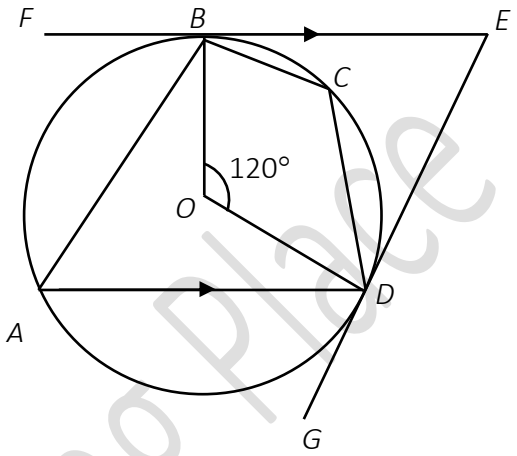
Equity Learning Place

Elementary Math Topical (Properties of Circle)

Question 3:

ABCD is a circle with centre *O*. *FE* and *GE* are tangents to the circle at *B* and *D* respectively. *AD* and *FE* are parallel and angle $BOD = 120°$.

Find, giving reasons for each answer,



a) angle *BED*,

b) angle *BAD*,

c) angle *BCD*,

d) angle *OBD*,

e) angle *ADO*.

f) Explain why *OBED* lies on the circumference of another circle.

a)

$$\angle BED = 360 - 90 - 90 - 120 = 60° \text{ (sum of angle in quadrilateral)}$$

b)

$$\angle BAD = 120 \div 2 = 60° \text{ (angle at centre} = 2 \text{ angle at circumference)}$$

c)

$$\angle BCD = 180 - 60 = 120° \text{ (angle in opp. segment)}$$

d)

$$\angle OBD = \frac{180 - 120}{2} = 30° \text{ (sum of angle in triangle)}$$

e)

$$\angle ABF = 60° \text{ (alt angle)}$$

$$\angle ABE = 120° \text{ (angle on a st. line)}$$

$$\angle ADE = 120° \text{ (opp angle of a parallelogram)}$$

$$\angle ADO = 120 - 90 = 30° \text{ (tangent perpendicular to radius)}$$

f)

$$\angle BOD + \angle BED = 180°$$

By property, angle in opposite segment, $OBED$ lies on the circumference of another circle.

Equity Learning Place

Elementary Math Topical (Properties of Circle)

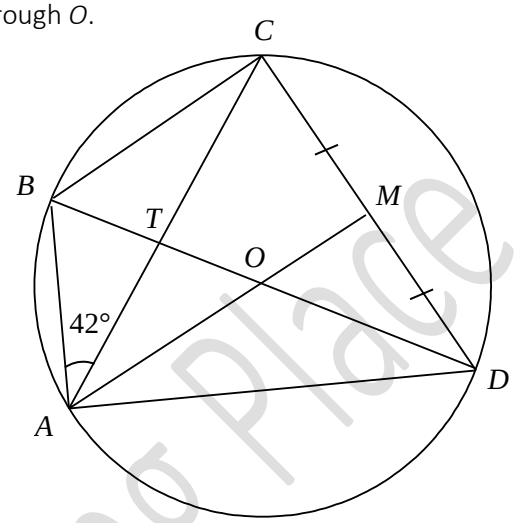
Question 4:

The diagram shows a circle *ABCD*, centre *O*.

M is the midpoint of the chord *CD* and *MA* passes through *O*.

T is the point of intersection of *AC* and diameter *BD*.

Angle $BAC = 42°$.



Find, giving reasons for each answer,

a) angle *BDC*,

b) angle *CBD*,

c) angle *AOD*,

d) angle *CTD*.

a)

$$\angle BDC = 42° \text{ (angle in same segment)}$$

b)

$$\angle BCD = 90° \text{ (right angle in semi circle)}$$

$$\angle CBD = 180 - 90 - 42 = 48° \text{ (sum of angle in triangle)}$$

c)

$$\angle CAD = 90 - 42 = 48° \text{ (right angle in semi circle)}$$

Triangle CAD is an isosceles triangle because $\angle AMD = 90°$, perpendicular bisector of chord will pass through the centre of the circle.

$$\angle CAM = 48 \div 2 = 24° \text{ (isosceles triangle)}$$

$$\angle ACM = 180 - 90 - 24 = 66° \text{ (sum of angle in triangle)}$$

$$\angle AOD = 66 \times 2 = 132° \text{ (angle at center} = 2 \text{ angle at circumference)}$$

d)

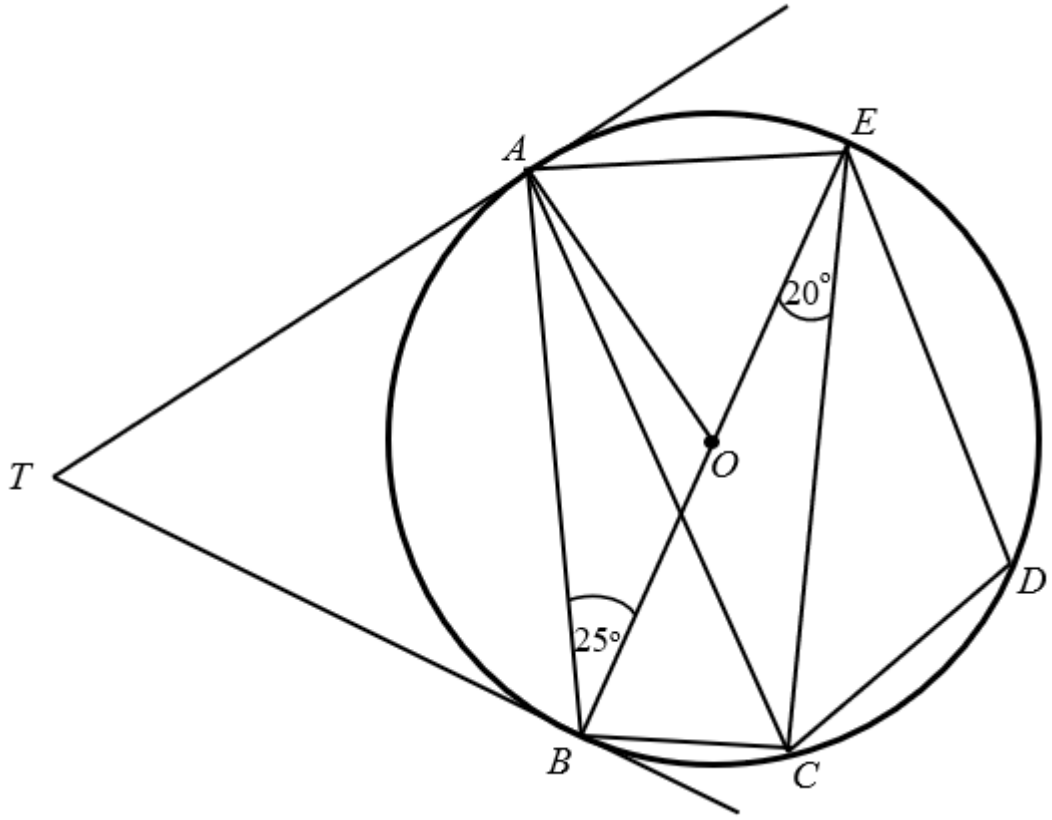
$$\angle ABD = 66° \text{ (angle in same segment)}$$

$$\angle BTA = 180 - 66 - 42 = 72° \text{ (sum of angle in triangle)}$$

$$\angle CTD = 72° \text{ (vert. opp. angle)}$$

Equity Learning Place

Question 5:



In the diagram, *O* is the centre of the circle. *A*, *B*, *C*, *D* and *E* are points on the circle. *TA* and *TB* are tangents to the circle. Angle *OBA* = 25° and angle *OEC* = 20°. Find, stating your reasons clearly,

a) angle *AOE*,

b) angle *CBE*,

c) angle *CDE*,

d) angle *OAC*.

e) angle *ATB*.

f) Explain why a circle may be drawn to pass through *A*, *O, B* and *T*.

a)

$$\angle AOE = 50° \text{ (angle at centre} = 2 \text{ angle at circumference)}$$

b)

$$\angle BCE = 90° \text{ (right angle in semi circle)}$$

$$\angle CBE = 180 - 90 - 20 = 70° \text{ (sum of angle in triangle)}$$

c)

$$\angle CDE = 180 - 70 = 110° \text{(angle in opp. segment)}$$

d)

$$\angle EAO = \frac{180 - 50}{2} = 65° \text{ (sum of angle in triangle)}$$

$$\angle CAE = 180 - 110 = 70° \text{ (angle in opp segment)}$$

$$\angle OAC = 70 - 65 = 5°$$

e)

$$\angle ABT = 90 - 25 = 65° \text{ (tangent perpendicular to radius)}$$

$$\angle BTA = 180 - 65 - 65 = 50° \text{ (tangent at ext. pt.)}$$

f)

$$\angle OAT + \angle OBT = 180°$$

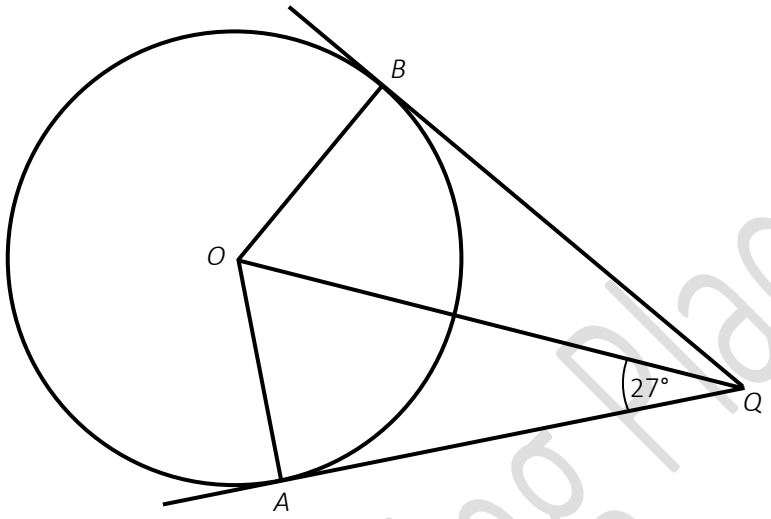
By property, angle in opposite segment, A, O, B and T are on the circumference of a circle.

Equity Learning Place

Elementary Math Topical (Properties of Circle)

Question 6:

The circle has centre O. QA and QB are tangents to the circle at A and B respectively, and $\angle OQA = 27°$.



a) Prove that triangles OAQ and OBQ are congruent.

b) Find $\angle BOQ$.

a)

$\angle OAQ = \angle OBQ = 90°$ (tangent perpendicular to radius)

$OA = OB$ (radius of circle)

$AQ = BQ$ (tangent at ext pt)

By SAS, triangle OAQ is similar to triangle OBQ.

b)

$\angle BOQ = 180 - 90 - 27 = 63°$ (sum of angle in triangle)

Equity Learning Place

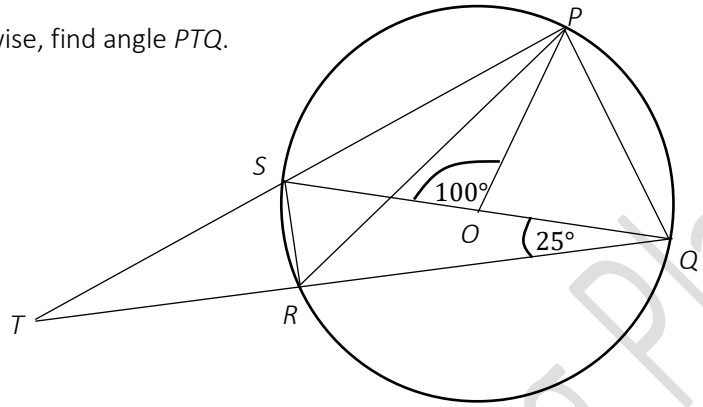
Elementary Math Topical (Properties of Circle)

Question 7:

P, *Q*, *R* and *S* lie on the circumference of a circle with centre *O*. *QS* is a straight line which passes through *O*. *PS* and *QR*, when produced, meet at point *T*.

a) Find angle *PRS* and angle *RPS*.

b) Hence or otherwise, find angle *PTQ*.



a)

$\angle PRS = 50°$ (angle at centre = 2 angle at circumference)

$\angle RPS = 25°$ (angle in same segment)

b)

$\angle PSR = 180 - 25 - 50 = 105°$ (sum of angle in triangle)

$\angle TSR = 180 - 105 = 75°$ (angle on a st. line)

$\angle SRT = 180 - 90 = 90°$ (angle on st. line)

$\angle PTQ = 180 - 75 - 90 = 15°$ (sum of angle in triangle)

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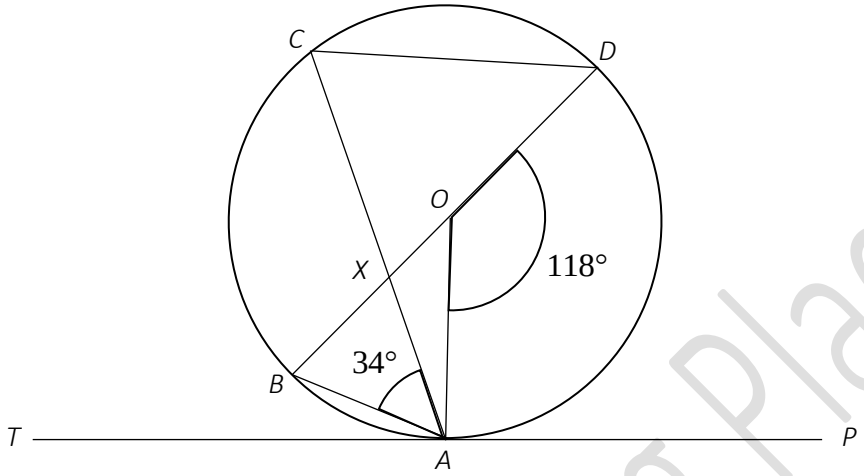
Elementary Math Topical (Properties of Circle)

Question 8:

In the figure, *BXOD* is a diameter of the circle with centre *O*. *TAP* is a tangent to the circle at *A*.



Angle *AOD* = $118°$ and angle *BAX* = $34°$.

Find, giving reasons for each answer,

a) angle *BDC*,

b) angle *ACD*,

c) angle *CXD*,

a)

$$\angle BDC = 34° \text{ (angle in same segment)}$$

b)

$$\angle ACD = 118 \div 2 = 59° \text{ (angle at centre} = 2 \text{ angle at circumference)}$$

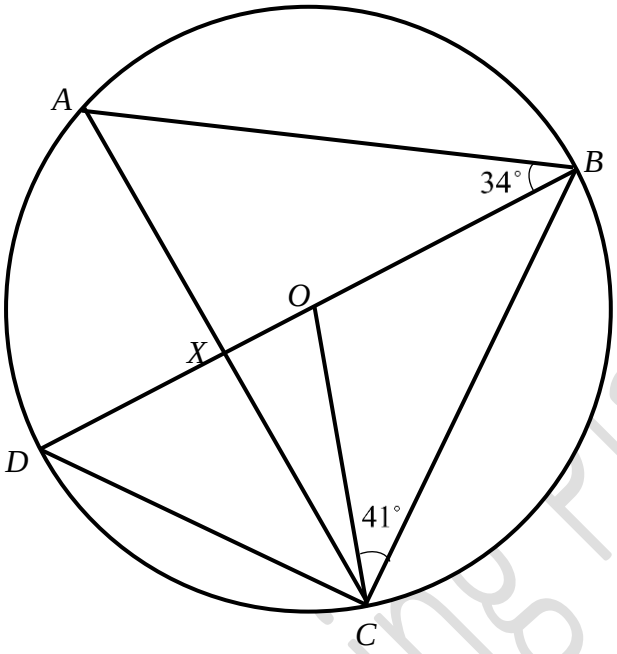
c)

$$\angle CXD = 180 - 59 - 34 = 87° \text{ (sum of angle in triangle)}$$

Equity Learning Place

Elementary Math Topical (Properties of Circle)

Question 9:



In the diagram, *O* is the centre of the circle. Lines *BD* and *AC* meet at *X*. Given angle *ABD* = 34° and angle *OCB* = 41°. By stating your reason(s) clearly, find

a) angle *ACD*

b) angle *BOC*

c) angle *CAB*

d) Show that triangles *ABX* and *DCX* are similar.

e) Given also that *AB* = 6.5 cm, *DC* = $4\frac{1}{3}$ cm and *DX* = 3 cm, calculate *AX*.

f) Find the values of $\frac{\text{Area of triangle } DCX}{\text{Area of triangle } ABX}$.

a)

$$\angle ACD = 34° \text{ (angle in same segment)}$$

b)

$$\angle OBC = 180 - 41 - 41 = 98° \text{ (sum of angle in triangle)}$$

c)

$$\angle CAB = 98 \div 2 = 49° \text{ (angle at centre = 2 angle at circumference)}$$

Equity Learning Place

d)

$\angle ABX = \angle DCX$ (angle in same segment)

$\angle AXB = \angle DXC$ (Vert. opp. angle)

By AA, triangle ABX is similar to triangle DCX

e)

$$\frac{AB}{DC} = \frac{AX}{DX}$$

$$\frac{6.5}{4\frac{1}{3}} = \frac{AX}{3}$$

$$AX = 4.5 \text{ cm}$$

f)

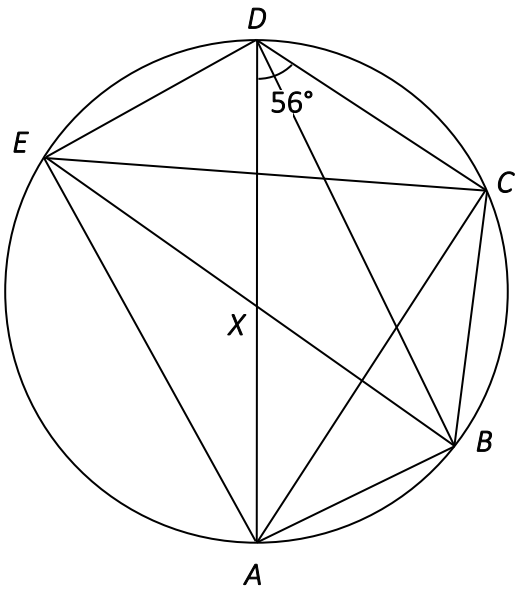
$$\frac{\text{Area } DCX}{\text{Area } ABX} = \left(\frac{\left(4\frac{1}{3}\right)}{6.5}\right)^2 = \frac{4}{9}$$

Equity Learning Place

Elementary Math Topical (Properties of Circle)

Question 10:

The points *A*, *B*, *C*, *D* and *E* lie on a circle with centre *X*. *AD* intersects *EB* at *X*. *DB* bisects angle *ADC*. Angle *ADC* = 56°.



Giving your reasons, write down

a) angle *DCA*

b) angle *DAC*

c) angle *CBA*

d) angle *AEB*

e) Triangle *BDX* is a special type of triangle. Name the triangle.

f) Find angle *AXB*

g) *Y* is a point that is not shown inside the diagram. Given that angle *AYB* is 40°, comment whether *Y* lies is inside the circle or outside the circle. Show your reasons clearly.

a)

$$\angle DCA = 90° \text{ (right angle in semi circle)}$$

b)

$$\angle DAC = 180 - 56 - 90 = 34° \text{ (Sum of angle in triangle)}$$

Equity Learning Place

c)

$\angle CBA = 180 - 56 = 124°$ (angle in opp seg.)

d)

$\angle AEB = 28°$ (DB bisect ADC, angle in same segment)

e)

Isosceles Triangle

f)

$\angle AXB = 28 \times 2 = 56°$ (angle at centre $= 2$ angle at circumference)

g)

Y is inside the circle, if Y is on the circle, by property,

angle at centre $= 2$ angle at circumference,

then $\angle AYB = 28°$. Since $\angle AYB = 40° > 28°$ it will be inside the circle.

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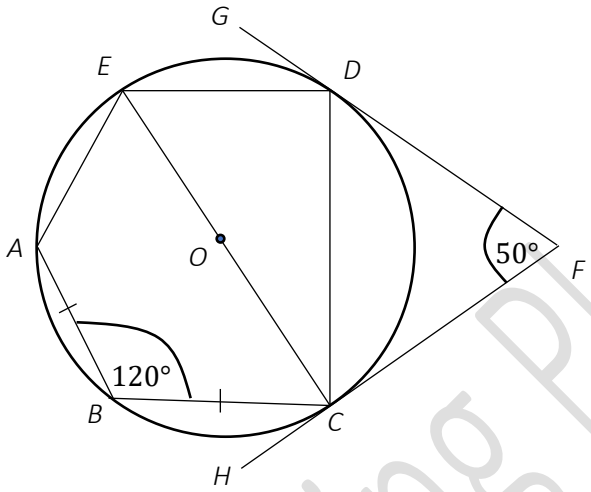
Elementary Math Topical (Properties of Circle)

Question 11:

In the diagram below, the points *A*, *B*, *C*, *D* and *E* lie on the circumference of a circle with centre *O*. *CE* is the diameter of the circle, angle $ABC = 120°$ and angle $DFC = 50°$. *GDF* and *HCF* are tangents to the circle at *D* and *C* respectively.



Giving your reasons, find

a) angle *ABE*,

b) angle *ADC*,

c) angle *AOC*,

d) angle *COF*

e) angle *CDF*

f) John claimed that tangent lines drawn from any 2 points that lie on the circumference of the circle will always meet. Do you agree? Explain.

a)

$$\angle BAC = \frac{180 - 120}{2} = 30° \text{ (Sum of angle in triangle)}$$

$$\angle EAC = 90° \text{ (right angle in semi circle)}$$

$$\angle EAB = 120°$$

$$\angle ABE = \frac{180 - 120}{2} = 30° \text{ (sum of angle in triangle)}$$

b)

$$\angle ADC = 180 - 120 = 60° \text{ (angle in opp. segment)}$$

Equity Learning Place

c)

$$\angle AOC = 60 \times 2 = 120° \text{ (angle at centre} = 2 \text{ angle at circumference)}$$

d)

$$\angle CFO = 50 \div 2 = 25° \text{ (Tangent at external point)}$$

$$\angle OCF = 90° \text{ (tangent perpendicular to radius)}$$

$$\angle COF = 180 - 90 - 25 = 65° \text{ (sum of angle in triangle)}$$

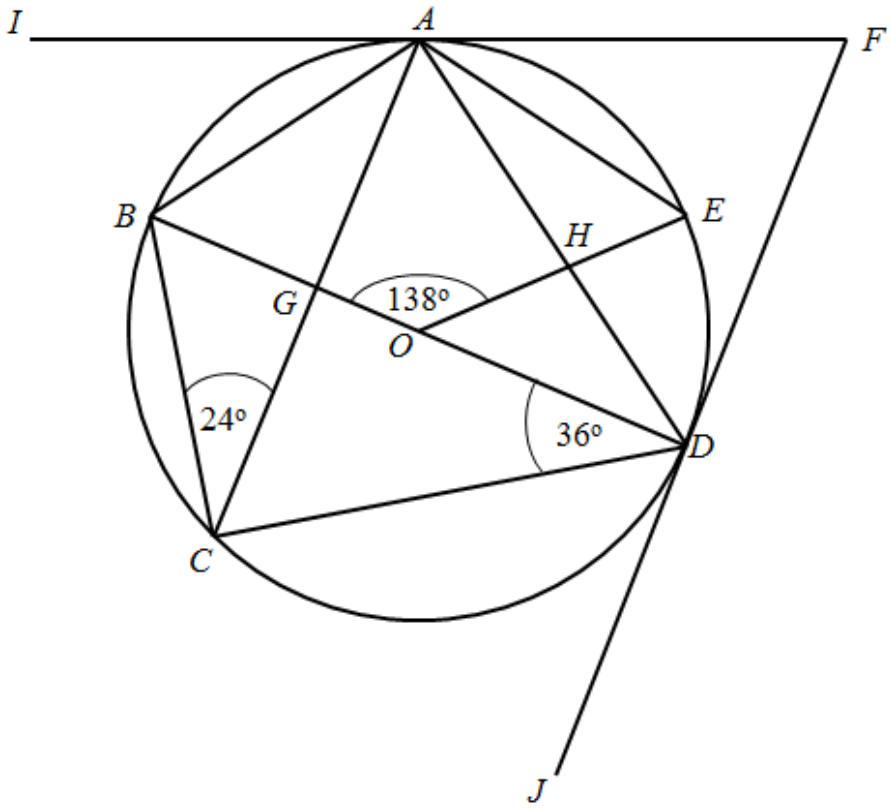
e)

$$\angle CDF = \frac{180 - 50}{2} = 65° \text{ (tangent at ext. point)}$$

f) No, if the tangent lines are parallel, they will not meet.

Equity Learning Place

Question 12:



Find, giving reasons for each answer,

a) $\angle BDA$

b) $\angle CBA$

c) $\angle ABD$

d) Prove that $\angle HAE = 21^o$

e) Explain why *BD* is not parallel to *AE*.

f) Find $\angle AFD$, giving the reason for your answer

Equity Learning Place

a)

$$\angle BDA = 24° \text{ (angle in same segment)}$$

b)

$$\angle CBA = 180 - 24 - 36 = 120° \text{ (angle in opp. segment)}$$

c)

$$\angle ABD = 180 - 90 - 24 = 66° \text{ (sum of angle in triangle)}$$

d)

$$\angle HOD = 180 - 138 = 42° \text{ (angle on a st. line)}$$

$$\angle HAE = 42 \div 2 = 21° \text{ (angle at centre} = 2 \text{ angle at circumference)}$$

e)

$$\text{Reflex } \angle BOE = 360 - 108 = 252° \text{ (angle at a point)}$$

$$\angle BAE = 252 \div 2 = 126° \text{ (angle at centre} = 2 \text{ angle at circumference)}$$

$$\angle ABD + \angle BAE = 192° \neq 180°$$

By property, interior angle, BD is not parallel to AE.

f)

$$\angle ADF = 90 - 24 = 67° \text{ (tangent perpendicular to radius)}$$

$$\angle AFD = 180 - 67 - 67 = 46° \text{ (tangent at ext. point)}$$

Equity Learning Place

Elementary Math Topical (Properties of Circle)

Question 13:

In the diagram, *O* is the centre of the circle *ABCD* and *AC* is a diameter. *TP* and *TQ* are the tangents to the circle at *A* and *B* respectively. $\angle ATB = 48º$ and $\angle CAD = 37º$.

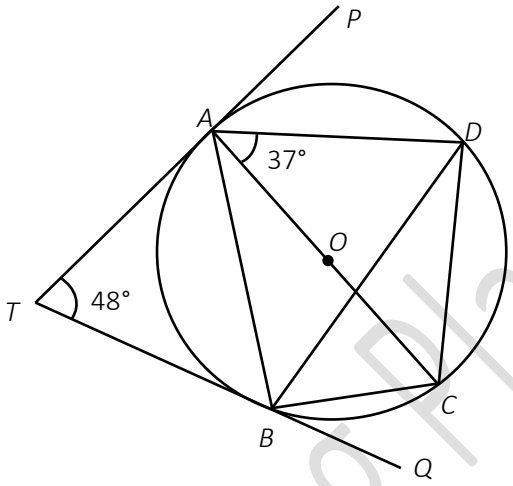
Find

a) $\angle ADC$

b) $\angle TAB$

c) $\angle BAC$

d) $\angle DBC$



a)

$$\angle ADC = 90° \text{ (right angle in semi circle)}$$

b)

$$\angle TAB = \frac{180 - 48}{2} = 66° \text{ (tangent at ext. pt.)}$$

c)

$$\angle BAC = 90 - 66 = 24° \text{ (tangent perpendicular to radius)}$$

d)

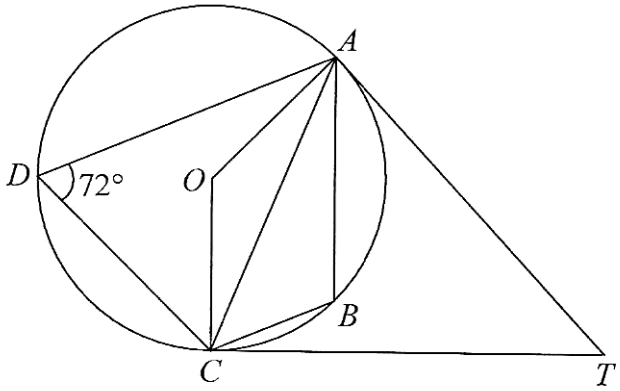
$$\angle DBC = 37° \text{ (angle in same segment)}$$

Equity Learning Place

Elementary Math Topical (Properties of Circle)

Question 14:

In the diagram below, *A*, *B*, *C* and *D* lie on the circumference of a circle with centre *O*. The tangents at *A* and *C* meet at *T* and $\angle ADC = 72°$.



Find, giving your reasons for your answer,

a) $\angle AOC$

b) $\angle ABC$

c) $\angle ATC$

a)

$$\angle AOC = 72 \times 2 = 144° \text{ (angle at centre} = 2 \text{ angle at circumference)}$$

b)

$$\angle ABC = 180 - 72 = 108° \text{ (angle in opp. segment)}$$

c)

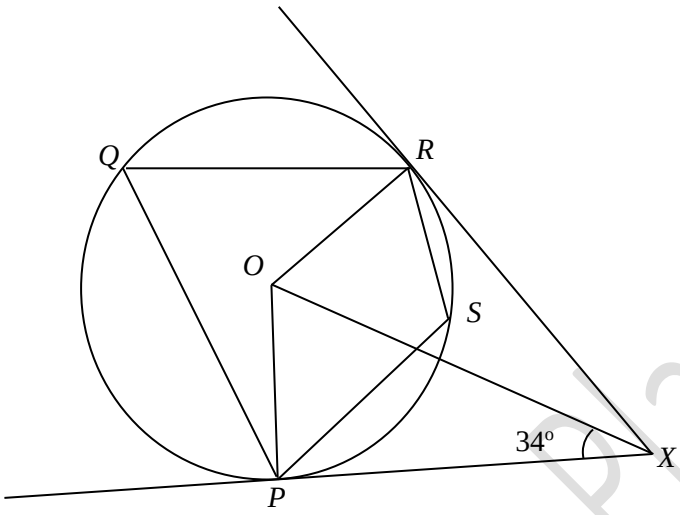
$$\angle OCA = \frac{180 - 144}{2} = 18° \text{ (sum of angle in triangle)}$$

$$\angle ACT = 90 - 18 = 72° \text{ (tangent perpendicular to radius)}$$

$$\angle ATC = 180 - 72 - 72 = 36° \text{ (tangent at ext. pt)}$$

Equity Learning Place

Question 15:



The diagram shows a circle *PQRS*, with centre *O*. *XP* and *XS* are tangents to the circle. Angle *PXO* = 34°.

a) Show that ΔOXP and ΔOXR are congruent triangles.

Stating the reasons, find

b) angle POR

c) angle PSR

d) It is given that angle $PYR = 56°$, where point *Y* is on the same side of *PR* as point *Q*. Does the point *Y* lie on the circumference of the circle, the centre of the circle, inside the circle or outside the circle? Give a reason for your answer.

a)

$\angle OPX = \angle ORX = 90°$ (tangent perpendicular to radius)

$OP = OR$ (radius of circle)

$PX = RX$ (Tangent at ext. pt.)

By SAS, triangle OXP is congruent to triangle OXR

Equity Learning Place

b)

$\angle POR = 360 - 90 - 90 - 34 - 34 = 112°$ (sum of angle in quadrilateral)

c)

Reflex $\angle POR = 360 - 112 = 248°$ (angle at a point)

$\angle PSR = 248 \div 2 = 124°$ (angle at centre $=$ 2 angle at circumference)

d)

$\angle PQR = 112 \div 2 = 56°$ (angle at centre $=$ 2 angle at circumference)

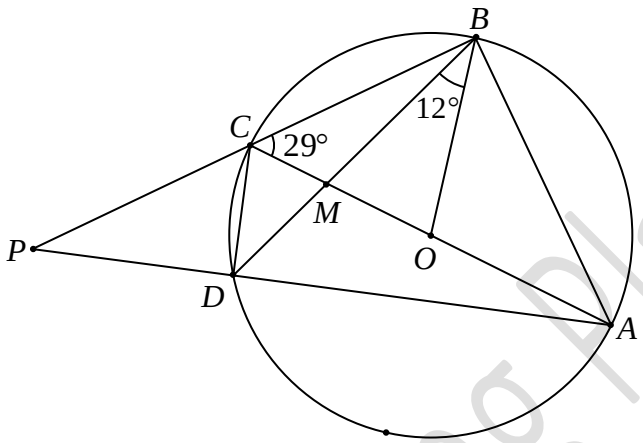
since $\angle PYR = \angle PQR = 56°$, Y lies on the circumference of the circle

Equity Learning Place

Elementary Math Topical (Properties of Circle)

Question 16:

The diagram shows a circle with centre *O* and *A*, *B*, *C* and *D* are points on the circle. *AOC*, *BMD* and *CMA* are straight lines. *BC* produced and *AD* produced meet at point *P*. $\angle OBM = 12°$ and $\angle BCM = 29°$.



Find, stating your reasons clearly,

a) $\angle BAC$

b) $\angle BAD$

c) $\angle CPD$

d) $\angle ACD$

a)

$$\angle BAC = 180 - 90 - 29 = 61° \text{ (sum of angle in triangle)}$$

b)

$$\angle CBM = 29 - 12 = 17° \text{ (isosceles triangle)}$$

$$\angle DAM = 17° \text{ (angle in same segment)}$$

$$\angle BAD = 17 + 61 = 78°$$

c)

$$\angle PCA = 180 - 29 = 151° \text{ (Angle on a st. line)}$$

$$\angle CPD = 180 - 151 - 17 = 12° \text{ (sum of angle in triangle)}$$

Equity Learning Place

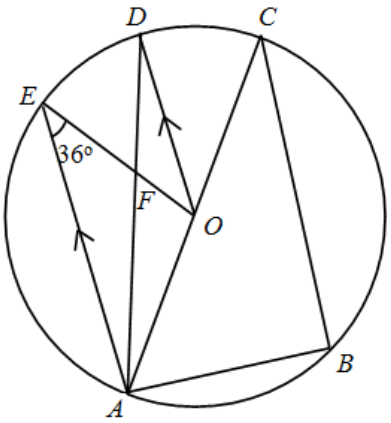
d)

$$\angle DCB = 180 - 78 = 102° \text{ (angle in opp. segment)}$$

$$\angle ACD = 102 - 29 = 80°$$

Question 17:

The points *A*, *B*, *C*, *D* and *E* lie on a circle with centre *O* and diameter *AC*. The angle *AEO* = 36° and *AE* is parallel to *OD*. *AD* intersects *OE* at *F*.



Calculate, showing your working clearly with stated reasons,

a) $\angle ABC$

b) $\angle COE$

c) $\angle DFE$

a)

$$\angle ABC = 90° \text{ (right angle in semi circle)}$$

b)

$$\angle EOA = 180 - 36 - 36 = 108° \text{ (sum of angle in triangle)}$$

$$\angle COE = 180 - 108 = 72° \text{ (angle on a st. line)}$$

Equity Learning Place

c)

$\angle DOF = 36°$ (alt angle)

$\angle FAE = 36 \div 2 = 18°$ (angle at centre = 2 angle at circumference)

$\angle AFE = 180 - 18 - 36 = 126°$ (sum of angle in triangle)

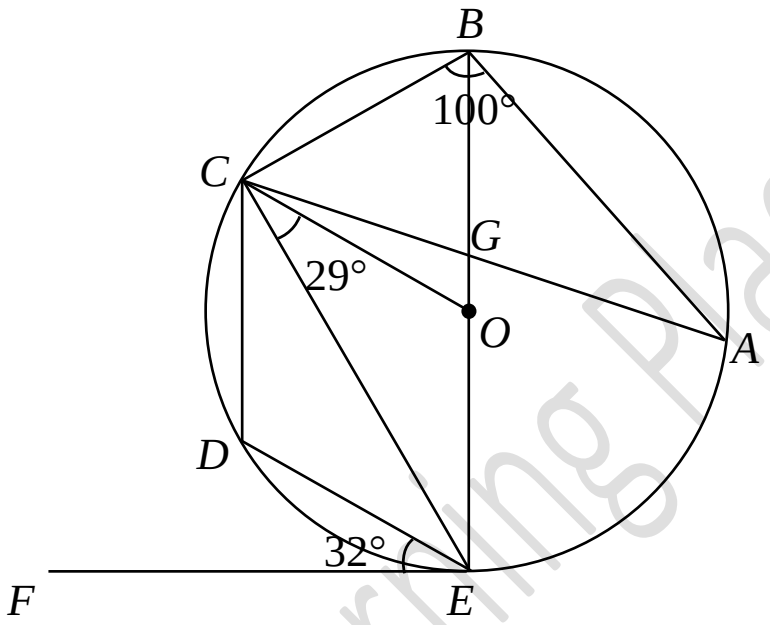
$\angle DFE = 180 - 126 = 54°$ (angle on a st. line)

Equity Learning Place

Elementary Math Topical (Properties of Circle)

Question 18:

In the diagram, *O* is the centre of the circle and *EF* is the tangent to the circle at *E*. The chord *AC* and the diameter *BE* intersect at *G*, $\angle ABC = 100°$, $\angle OCE = 29°$ and $\angle DEF = 32°$.



a) Showing all reasons clearly, find $\angle CED$.

b) Explain why *CO* is parallel to *DE*.

c) find $\angle CDE$,

d) find $\angle ACO$.

a)

$$\angle CEO = 29° \text{ (isosceles triangle)}$$

$$\angle CED = 90 - 32 - 29 = 29° \text{ (tangent perpendicular to radius)}$$

b)

$$\angle OCE = \angle CED = 29°$$

By property, alternate angle, CO is parallel to DE.

c)

$$\angle CDE = 180 - 29 - 29 = 122° \ (COED \text{ is a rhombus})$$

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d)

Reflex $\angle COA = 100 \times 2 = 200°$ (angle at centre = 2 angle at circumference)

$\angle COA = 360 - 200 = 160°$ (Angle at a point)

$\angle ACO = \frac{180 - 160}{2} = 10°$ (Sum of angle in triangle)

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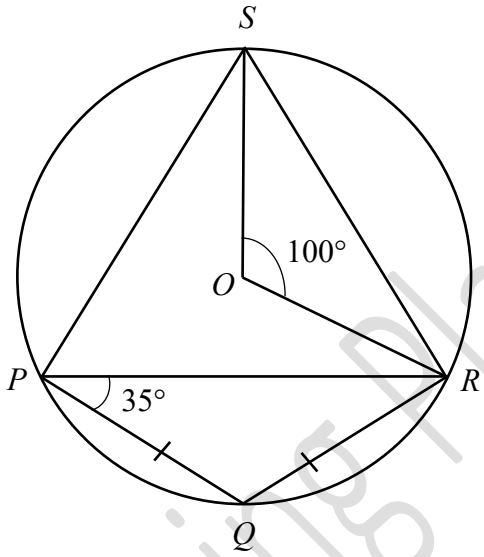
Elementary Math Topical (Properties of Circle)

Question 19:

P, Q, R and *S* are points on a circle, with centre *O*. It is given that *PQ* = *QR*, $\angle QPR = 35°$ and $\angle SOR = 100°$.



a) Calculate $\angle PQR$.

b) Find $\angle PSR$. Give a reason for each step of your working.

c) Find $\angle SPR$. Give a reason for each step of your working.

d) Explain why triangle *SRQ* is not a right-angled triangle.

a)

$$\angle PQR = 180 - 35 - 35 = 110° \text{ (sum of angle in a triangle)}$$

b)

$$\angle PSR = 180 - 110 = 70° \text{ (angle in opp. seg.)}$$

c)

$$\angle SPR = 50° \text{ (angle at centre} = 2 \text{ angle at circumference)}$$

d)

$$\angle SPQ = 35 + 70 = 105°$$

$$\angle SRQ = 180 - 105 = 75° \text{ (Angle in opp. seg)}$$

$$\text{Since } \angle SRQ \neq 90°, \text{triangle } SRQ \text{ is not a right} - \text{angled triangle}$$

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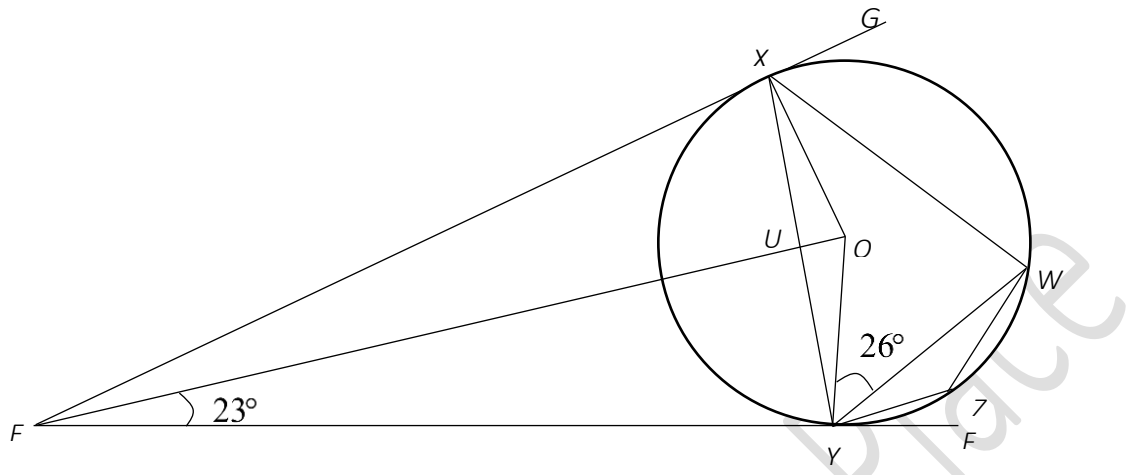
Elementary Math Topical (Properties of Circle)

Question 20:



The diagram shows a circle *WXYZ*, with centre *O*. *EYF* and *EXG* are tangents to the circle and *U* is the point of intersection of *XY* and *OE*. $\angle YEO = 23°$ and $\angle WYO = 26°$

a) Name a pair of congruent triangles.

Find

b) $\angle YOE$

c) $\angle OXY$

d) $\angle YXW$

e) $\angle YZW$

f) A point *V* is to be marked on the diagram on the same side of *XY* as *W* such that $\angle YVX = 56°$. Does the point *V* lie inside the circle, outside the circle or on the circumference of the circle? Explain your answer.

a) Triangle XOE and triangle YOE

b)

$$\angle YOE = 180 - 90 - 23 = 67° \text{ (sum of angle in triangle)}$$

c)

$$\angle XOY = 67 \times 2 = 134° \text{ (tangent at ext. pt)}$$

$$\angle OXY = \frac{180 - 134}{2} = 23° \text{ (Sum of angle in triangle)}$$

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d)

$\angle YWX = 134 \div 2 = 67°$ (angle at centre = 2 angle at circumference)

$\angle YXW = 180 - 67 - 26 - 23 = 64°$ (sum of angle in triangle)

e)

$\angle YZW = 180 - 64 = 116°$ (sum of angle in triangle)

f)

$\angle YWX = 67°$

since $\angle YVX = 56° < 67°, V$ will be outside of the circle.

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