

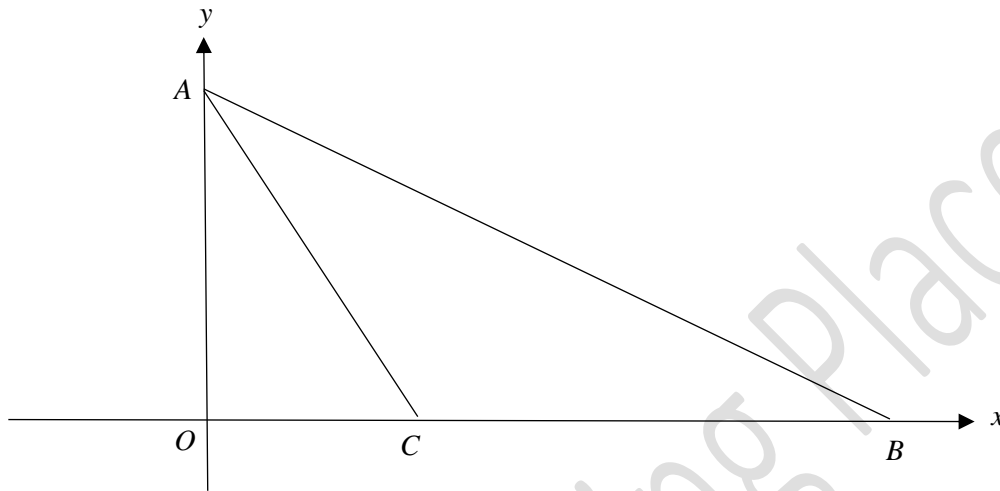
# EQUITY

## LEARNING PLACE

### Elementary Math Topical (Coordinate Geometry)

#### Question 1:

The diagram shows a sketch of a straight line  $AB$  with equation  $5y + 2x = 35$ . The length of  $BC$  is 10 units.



- Show that the coordinates of A and B are  $(0,7)$  and  $(17.5,0)$  respectively.
- Find the length of  $AB$ .
- Find the equation of the line  $AC$ .
- Find the shortest distance from  $C$  to the line  $AB$ .
- The line  $y = 2x - 1$  intersects  $AB$  at  $D$ . Find the coordinates of  $D$ .

a)

$$5y + 2x = 35$$

$$\text{when } x = 0,$$

$$5y = 35$$

$$y = 7$$

$$\therefore A(0, 7)$$

$$\text{when } y = 0$$

$$2x = 35$$

$$x = 17.5$$

$$\therefore B(17.5, 0)$$

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## LEARNING PLACE

### Elementary Math Topical (**Coordinate Geometry**)

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b)

$$\text{Length } AB = \sqrt{17.5^2 + 7^2} = 18.848 \text{ units} = 18.8 \text{ units}$$

c)

$$C(7.5, 0)$$

$$\text{Grad } AC = \frac{7 - 0}{0 - 7.5} = -\frac{14}{15}$$

Eqn of AC:

$$y = -\frac{14}{15}x + 7$$

d)

$$\frac{1}{2}(10)(7) = \frac{1}{2}(x)(18.848)$$

$$x = 3.71 \text{ units}$$

e)

Sub  $y = 2x - 1$  into equation of AB

$$5(2x - 1) + 2x = 35$$

$$10x - 5 + 2x = 35$$

$$12x = 40$$

$$x = \frac{10}{3}$$

$$y = \frac{17}{3}$$

$$D\left(\frac{10}{3}, \frac{17}{3}\right)$$

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### Elementary Math Topical (Coordinate Geometry)

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#### Question 2:

The point  $P$  is  $(0, k - 3)$  and it lies on the line,  $l$ , with gradient =  $-\frac{2}{5}$  and  $y$ -intercept = 2.

a) Find the value of  $k$ .

b) Given that  $Q(-2, n)$  lies on the line  $l$ , find the value of  $n$ .

a)

$$\text{Eqn of } l: y = -\frac{2}{5}x + 2$$

$$k - 3 = 2$$

$$k = 5$$

b) sub  $x = -2$  and  $y = n$

$$n = -\frac{2}{5}(-2) + 2$$

$$n = \frac{6}{5}$$

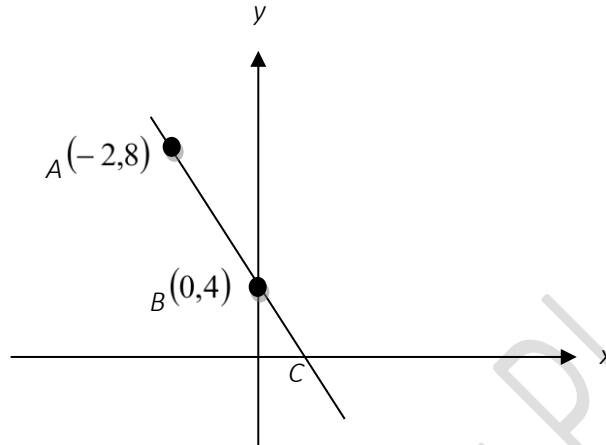
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### Elementary Math Topical (Coordinate Geometry)

#### Question 3:

The diagram shows a straight line passing through the points  $A(-2,8)$ ,  $B(0,4)$  and  $C$ .



a) Calculate the length of the line joining  $A$  to  $B$ .

b) Find the equation of the line.

c) Write down the coordinates of point  $C$ .

a)

$$\text{Length } AB = \sqrt{(-2 - 0)^2 + (8 - 4)^2} = 4.47 \text{ units}$$

b)

$$\text{Grad } AB = \frac{8 - 4}{-2 - 0} = -2$$

Eqn of  $AB$

$$y = -2x + 4$$

c)

when  $y = 0$

$$0 = -2x + 4$$

$$2x = 4$$

$$x = 2$$

$$C(2, 0)$$

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### Elementary Math Topical (Coordinate Geometry)

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#### Question 4:

The coordinates of points  $A$  and  $B$  are  $(-2, 5)$  and  $(2.5, -4)$  respectively.

- a) Calculate the distance between the two points.  
b) If a line passes through the two points, find the equation of the line.

a)

$$\text{Length } AB = \sqrt{(-2 - 2.5)^2 + (5 - (-4))^2} = 10.1 \text{ units}$$

b)

$$\text{Grad } AB = \frac{5 - (-4)}{-2 - 2.5} = -2$$

Eqn of  $AB$

$$-4 = (2.5)(-2) + c$$

$$c = 0.5$$

$$y = -2x + \frac{1}{2}$$

#### Question 5:

Line  $p$  passes through the points  $A(-1, -6)$  and  $B(3, 4)$ .

- a) Find the length of  $AB$ .  
b) Find the gradient of the line  $p$ .  
c) Find the equation of line  $p$ .  
d) Verify if the point  $(-1, 27)$  lies on the line  $p$ .

a)

$$\text{Length } AB = \sqrt{(3 - (-1))^2 + (4 - (-6))^2} = 10.8 \text{ units}$$

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### Elementary Math Topical (Coordinate Geometry)

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b)

$$\text{Grad } AB = \frac{4 - (-6)}{3 - (-1)} = \frac{5}{2}$$

c)

Eqn of  $p$ :

$$4 = \left(\frac{5}{2}\right)(3) + c$$

$$c = -\frac{7}{2}$$

$$y = \frac{5}{2}x - \frac{7}{2}$$

d) Let  $x = -1$

$$y = -\frac{5}{2} - \frac{7}{2} = -6$$

The point does not lie on the line since the line passes through  $(-1, -6)$ .

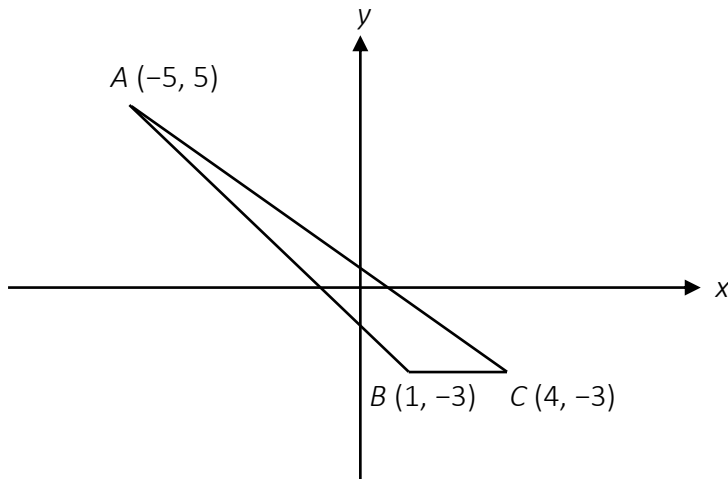
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### Elementary Math Topical (Coordinate Geometry)

#### Question 6:

In the diagram, points  $A(-5, 5)$ ,  $B(1, -3)$  and  $C(4, -3)$  form an obtuse triangle.



Find

- the length  $AB$ ,
- the coordinates of  $D$  such that  $ABCD$  is a parallelogram
- the equation of line  $AB$ ,
- the coordinates of  $M$  and of  $N$  given that  $AB$  cuts the  $x$ -axis and  $y$ -axis at  $M$  and  $N$  respectively,
- the area of triangle  $ABC$ ,
- the exact value of  $\cos ABC$ .

a)

$$\text{Length } AB = \sqrt{(5 - (-3))^2 + (-5 - 1)^2} = 10 \text{ units}$$

- b) By observation  $BC$  is horizontal with length 3 so  $AD$  is also horizontal with length 3

$$D(-2, 5)$$

c)

$$\text{Gradient of } AB = \frac{5 - (-3)}{-5 - 1} = -\frac{4}{3}$$

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### Elementary Math Topical (Coordinate Geometry)

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Eqn  $AB$

$$-3 = -\frac{4}{3}(1) + c$$

$$c = -\frac{5}{3}$$

$$y = -\frac{4}{3}x - \frac{5}{3}$$

d) when  $x = 0$

$$y = -\frac{5}{3}$$

$$N\left(0, -\frac{5}{3}\right)$$

When  $y = 0$

$$\frac{4}{3}x = \frac{5}{3}$$

$$x = \frac{5}{4}$$

$$M\left(\frac{5}{4}, 0\right)$$

e)

$$\text{Area } ABC = \frac{1}{2}(3)(8) = 12 \text{ units}^2$$

f)

$$\cos ABC = -\frac{3}{5}$$

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### Elementary Math Topical (**Coordinate Geometry**)

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#### Question 7:

The line  $6x - 3y = 24$  cuts the  $x$ -axis at  $M$  and  $y$ -axis at  $N$ .

a) Find the coordinates of  $M$  and  $N$ .

b) Another line  $l$  has the same gradient as the given line and passes through the point  $(1, 5)$ . Find the equation of the line  $l$ .

a) when  $x = 0$

$$-3y = 24$$

$$y = -8$$

$$N(0, -8)$$

When  $y = 0$ ,

$$6x = 24$$

$$x = 4$$

$$M(4, 0)$$

b)

$$-3y = -6x + 24$$

$$y = 2x - 8$$

$$\text{Gradient} = 2$$

Eqn of  $l$ :

$$5 = (2)(1) + c$$

$$c = 3$$

$$y = 2x + 3$$

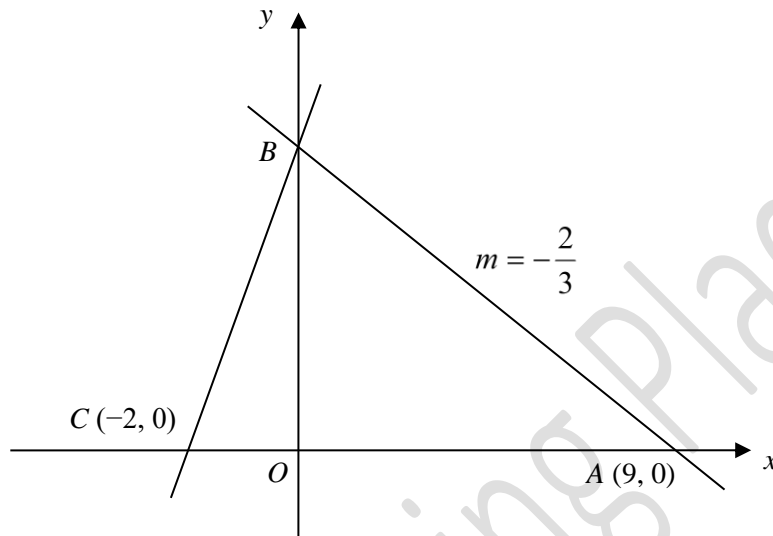
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### Elementary Math Topical (Coordinate Geometry)

#### Question 8:

In the diagram, which is not drawn to scale, the coordinates of A and C are (9, 0) and (-2, 0) respectively.



The gradient,  $m$ , of the line  $AB$  is  $-\frac{2}{3}$ .

- Find the equation of the line  $AB$ .
- Find the equation of the line  $BC$ .
- Determine whether triangle  $ABC$  is a right-angled triangle.
- Find the area of triangle  $ABC$ .

a)

Eqn of  $AB$

$$0 = -\frac{2}{3}(9) + c$$

$$c = 6$$

$$y = -\frac{2}{3}x + 6$$

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### Elementary Math Topical (Coordinate Geometry)

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b)

$$\text{Grad } BC = \frac{6 - 0}{0 - (-2)} = 3$$

Eqn  $BC$

$$y = 3x + 6$$

c)

$$\tan CBO = \frac{2}{6}$$

$$\angle CBO = 18.434$$

$$\tan OBA = \frac{3}{2}$$

$$\angle OBA = 26.56$$

Since  $\angle OBA + \angle CBO \neq 90^\circ$ , triangle  $ABC$  is not a right-angled triangle.

d)

$$\text{Area } ABC = \frac{1}{2}(11)(6) = 33 \text{ units}^2$$

Question 9:

A straight line passes through the points  $A(0, 16)$  and  $B(8, 2)$ .

a) Find the equation of line  $AB$ .

b) Find the value of  $n$  given that  $(6, n)$  lies on the line  $AB$ .

a)

$$\text{Grad } AB = \frac{16 - 2}{0 - 8} = -\frac{7}{4}$$

$$\text{eqn } AB: y = -\frac{7}{4}x + 16$$

b) Let  $x = 6, y = n$

$$n = -\frac{7}{4}(6) + 16$$

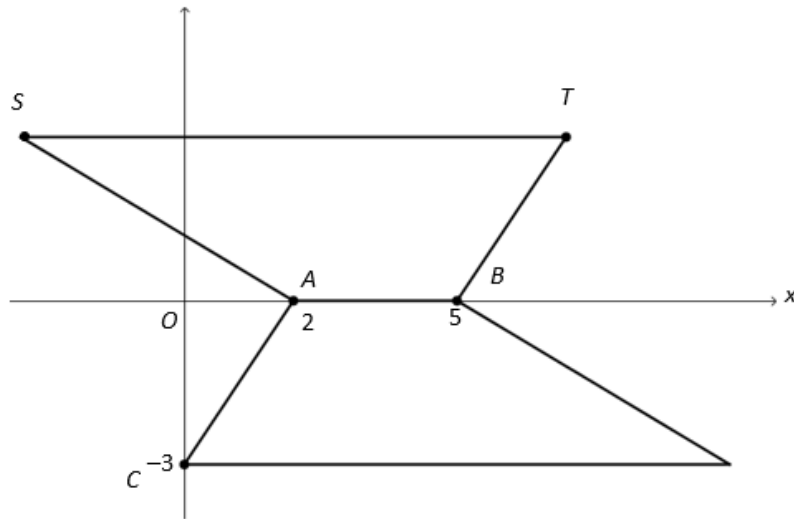
$$n = 5.5$$

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### Elementary Math Topical (Coordinate Geometry)

Question 10:



The diagram shows two congruent trapeziums on an  $xy$ -plane. The points  $A$  and  $B$  lie on the  $x$ -axis and the point  $C$  lies on the  $y$ -axis.

- a) Find the coordinates of the point  $T$ .
- b) Given that the combined area of both trapeziums is  $39 \text{ units}^2$ , find the coordinates of the point  $S$ .

a)

$$T(7, 3)$$

b)

$$\frac{1}{2}(3 + ST)(3) = \frac{1}{2}(39)$$

$$3 + ST = 13$$

$$ST = 10$$

$$S(-3, 3)$$

# EQUITY

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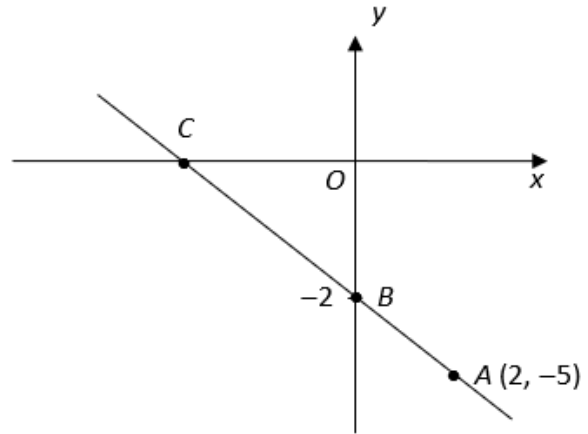
### Elementary Math Topical (Coordinate Geometry)

#### Question 11:

The diagram shows a sketch of a straight line passing through the points  $A(2, -5)$ ,  $B$  on the  $y$ -axis and  $C$  on the  $x$ -axis.

Find the

- equation of the line,
- coordinates of  $C$
- equation of the line parallel to  $BC$  and passing through the origin



a)

$$\text{Grad of } AB = \frac{-5 - (-2)}{2 - 0} = -\frac{3}{2}$$

$$\text{eqn of line: } y = -\frac{3}{2}x - 2$$

b)

when  $y = 0$

$$\frac{3}{2}x = -2$$

$$x = -\frac{4}{3}$$

c)

$$y = -\frac{3}{2}x$$

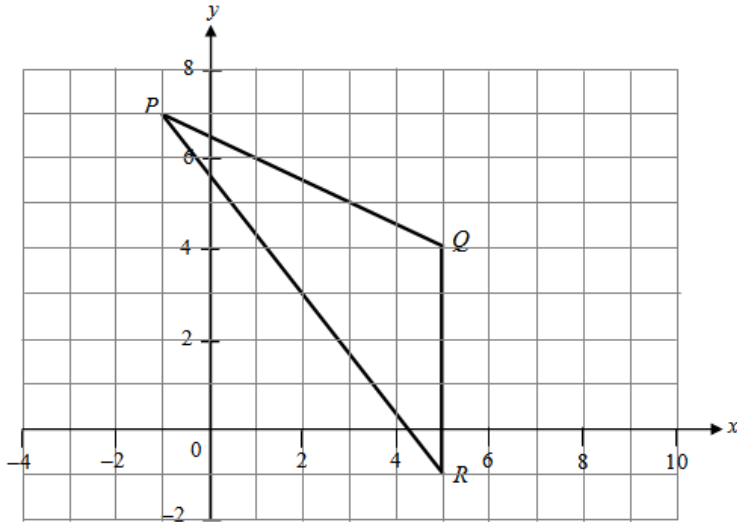
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### Elementary Math Topical (Coordinate Geometry)

#### Question 12:

The points  $P(-1, 7)$ ,  $Q(5, 4)$  and  $R(5, -1)$  are shown in the diagram



Find

- the gradient of line  $PQ$ ,
- the equation of the line  $PQ$ ,
- the length of line  $PQ$ ,
- $\cos \angle PQR$ ,
- the area of triangle  $PQR$ .

a)

$$\text{Grad of } PQ = \frac{7 - 4}{-1 - 5} = -\frac{1}{2}$$

b)

Eqn of  $PQ$ :

$$7 = \left(-\frac{1}{2}\right)(-1) + c$$

$$c = \frac{13}{2}$$

$$y = -\frac{1}{2}x + \frac{13}{2}$$

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### Elementary Math Topical (Coordinate Geometry)

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c)

$$\text{Length } PQ = \sqrt{(5 - (-1))^2 + (4 - 7)^2} = \sqrt{45} = 6.71 \text{ units}$$

d)

$$\cos PQR = -\frac{3}{\sqrt{45}}$$

e)

$$\text{Area of } PQR = \frac{1}{2}(5)(6) = 15 \text{ units}^2$$

#### Question 13:

The coordinates of  $O$ ,  $A$  and  $B$  are  $(0, 0)$ ,  $(5, 0)$  and  $(7, 8)$  respectively. Find

a) the gradient of  $AB$ ,

b) the equation of  $AB$

c) the area of triangle  $OAB$

d) The distance between the points  $(2 - k, 1)$  and  $(2, k)$  is  $\sqrt{5 - 4k}$  units.  
Find the possible values of  $k$ .

a)

$$\text{Grad } AB = \frac{8 - 0}{7 - 5} = 4$$

b)

eqn of  $AB$

$$0 = 5(4) + c$$

$$c = -20$$

$$y = 4x - 20$$

c)

$$\text{Area of triangle} = \frac{1}{2}(5)(8) = 20 \text{ units}^2$$

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## LEARNING PLACE

### Elementary Math Topical (**Coordinate Geometry**)

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d)

$$\sqrt{((2-k-2)^2 + (1-k)^2)} = \sqrt{5-4k}$$

$$k^2 + 1 - 2k + k^2 = 5 - 4k$$

$$2k^2 + 2k - 4 = 0$$

$$k^2 + k - 2 = 0$$

$$(k+2)(k-1) = 0$$

$$k = -2 \text{ or } k = 1$$

Question 14:

The line  $3y - 6x + 9 = 0$  cuts the  $y$ -axis at  $P$  and the  $x$ -axis at  $Q$ .

a) State the gradient of the line  $PQ$ .

b) Find the coordinates of  $P$  and  $Q$ .

c) Find the equation of another line,  $l$ , which is parallel to the line  $PQ$  and passes through the point  $(1, -8)$ .

a)

$$3y = 6x - 9$$

$$y = 2x - 3$$

$$\text{Gradient of } PQ = 2$$

b)

$$\text{when } x = 0$$

$$y = -3$$

$$P(0, -3)$$

$$\text{when } y = 0$$

$$x = \frac{3}{2}$$

$$Q\left(\frac{3}{2}, 0\right)$$

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### Elementary Math Topical (Coordinate Geometry)

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c)

eqn of line

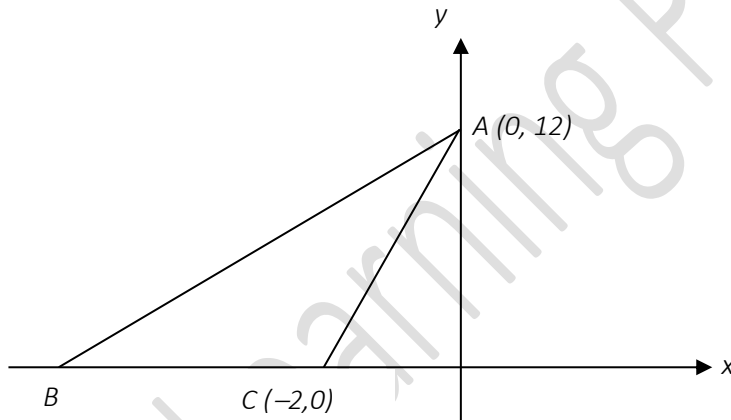
$$-8 = 1(2) + c$$

$$c = -10$$

$$y = 2x - 10$$

Question 15:

In the figure below,  $B$  lies on the  $x$ -axis. The area of  $\triangle ABC$  is 48 units<sup>2</sup> and the equation of the line  $AB$  is  $y = ax + b$ .



- Find the coordinates of the point  $B$ ,
- Find the value of  $a$  and of  $b$ .
- Show that the points  $B$ ,  $D$  and  $A$  are collinear, where  $D$  is  $(-5, 6)$ .

a)

$$\frac{1}{2}(BC)(12) = 48$$

$$BC = 8$$

$$B(-10, 0)$$

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### Elementary Math Topical (Coordinate Geometry)

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b)

$$\text{Grad } AB = \frac{12 - 0}{0 - (-10)} = \frac{6}{5}$$

$$\therefore a = \frac{6}{5}, \quad b = 12$$

c)

$$\text{Grad } AD = \frac{6 - 12}{-5 - 0} = \frac{6}{5}$$

Since the gradient is the same and  $A$  is a common point,  $B, D, A$  are collinear.

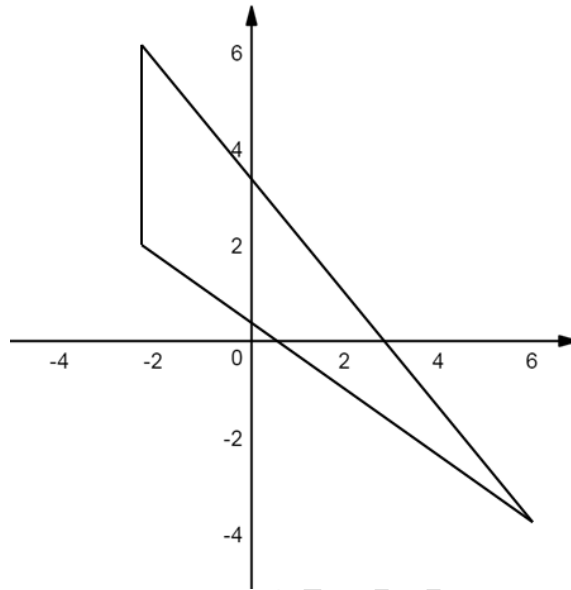
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### Elementary Math Topical (Coordinate Geometry)

#### Question 16:

The diagram below shows three labelled points  $A(-2, 6)$ ,  $B(-2, 2)$  and  $C(6, -4)$ .  $D$  is a point where the line  $AC$  meets the  $x$ -axis.



Find

- the gradient of the line  $AC$  and hence the equation of  $AC$
- the coordinates of  $D$  and **hence** the acute angle which the line  $AD$  makes with the  $x$ -axis,
- the **exact** value of  $\cos \angle ABC$
- the area of triangle  $ABC$

a)

$$\text{Grad } AC = \frac{6 - (-4)}{-2 - 6} = -\frac{5}{4}$$

Eqn of  $AC$

$$6 = -\frac{5}{4}(-2) + c$$

$$c = \frac{7}{2}$$

$$y = -\frac{5}{4}x + \frac{7}{2}$$

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### Elementary Math Topical (Coordinate Geometry)

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b) when  $y = 0$

$$\frac{5}{4}x = \frac{7}{2}$$

$$x = \frac{14}{5}$$

$$D\left(\frac{14}{5}, 0\right)$$

$$\tan \theta = \frac{5}{4}$$

$$\theta = 51.3^\circ$$

c)

$$\text{Length } BC = \sqrt{(-4 - 2)^2 + (6 - (-2))^2} = 10$$

$$\cos ABC = -\frac{3}{5}$$

d)

$$\text{Area of triangle } ABC = \frac{1}{2}(4)(8) = 16 \text{ units}^2$$

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### Elementary Math Topical (**Coordinate Geometry**)

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#### Question 17:

The line  $4x + 2y = 14$  crosses the  $x$ -axis at the point  $A$ .

- a) Find the gradient of the line.
- b) Find the coordinates of point  $A$ .

a)

$$2y = -4x + 14$$

$$y = -2x + 7$$

Gradient of line is 7

b) when  $y = 0$

$$4x = 14$$

$$x = \frac{7}{2}$$

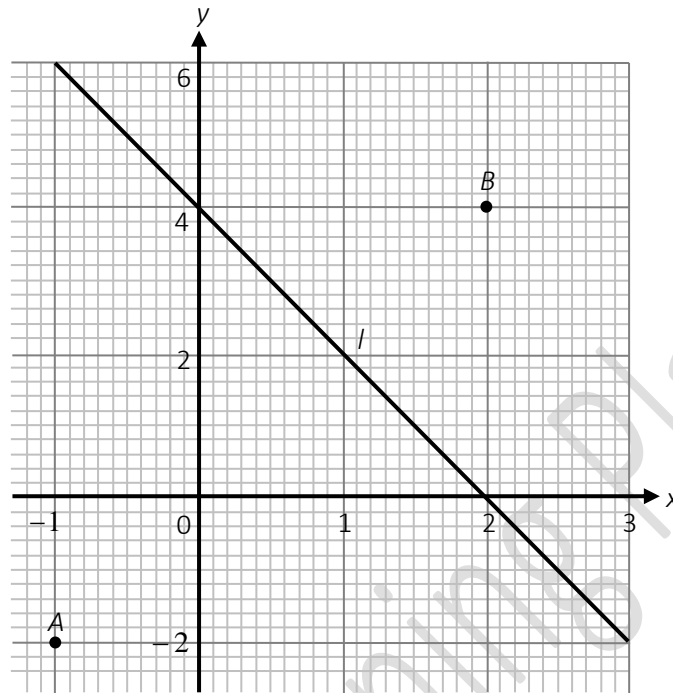
$$A\left(\frac{7}{2}, 0\right)$$

# EQUITY

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### Elementary Math Topical (Coordinate Geometry)

Question 18:



a) Calculate the length of  $AB$ .

b) Calculate the gradient of the line  $l$ .

c) Find the equation of line  $l$ .

a)

$$\text{Length } AB = \sqrt{(2 - (-1))^2 + (4 - (-2))^2} = 6.71 \text{ units}$$

b)

$$\text{Gradient} = \frac{4 - (-2)}{2 - (-1)} = 2$$

c)

Eqn of  $l$

$$4 = 2(2) + c$$

$$c = 0$$

$$y = 2x$$

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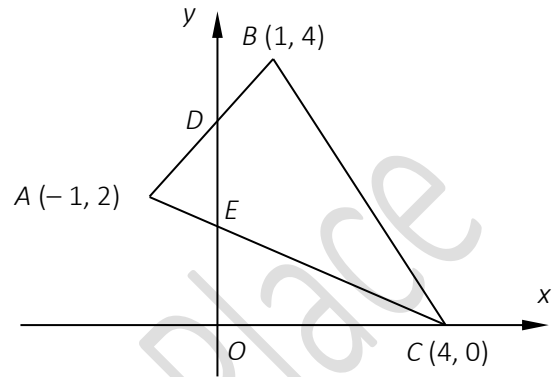
## LEARNING PLACE

### Elementary Math Topical (Coordinate Geometry)

Question 19:

In the diagram,  $O$  is the origin,  $A$  is  $(-1, 2)$ ,  $B$  is  $(1, 4)$ ,  $C$  is  $(4, 0)$ .  $D$  and  $E$  are points of intersections where  $AB$  and  $AC$  cut the  $y$ -axis.

- a) Find the length of  $BC$ .
- b) Determine the equation of line  $AC$ .
- c) State the value of  $\tan \angle ACO$  as a fraction.
- d) Calculate the value of  $\frac{\text{area of triangle } ADE}{\text{area of triangle } ABC}$ .



a)

$$BC = \sqrt{(4 - 1)^2 + (0 - 4)^2} = 5 \text{ units}$$

b)

$$\text{Gradient } AC = \frac{2 - 0}{-1 - 4} = -\frac{2}{5}$$

Eqn of  $AC$

$$0 = 4\left(-\frac{2}{5}\right) + c$$

$$c = \frac{8}{5}$$

$$y = -\frac{2}{5}x + \frac{8}{5}$$

c)

$$\tan \angle ACO = \frac{2}{5}$$

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### Elementary Math Topical (Coordinate Geometry)

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d)

$$\text{when } x = 0, y = \frac{8}{5}$$

$$D(0,3)$$

$$\frac{AD}{AB} = \frac{1}{2}$$

$$\frac{AE}{AC} = \frac{1}{4}$$

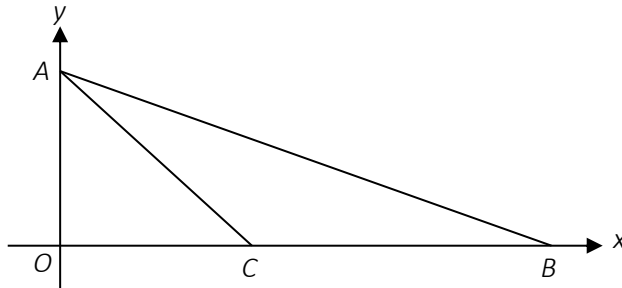
$$\begin{aligned}\frac{\text{Area of } ADE}{\text{Area of } ABC} &= \frac{\frac{1}{2}(AD)(AE) \sin \angle DAE}{\frac{1}{2}(AB)(AC) \sin \angle DAE} \\ &= \frac{(AD)(AE)}{(AB)(AC)} \\ &= \frac{1}{2} \left( \frac{1}{4} \right) \\ &= \frac{1}{8}\end{aligned}$$

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### Elementary Math Topical (Coordinate Geometry)

Question 20:



In the diagram, the equation of the line  $AB$  is  $3y + x = 18$  and the length of  $BC$  is 12 units. Write down

- a) the coordinates of  $A$
- b) the coordinates of  $B$
- c) Find the length of  $AB$ .

$Q$  is a point on the line  $AB$  with  $y$ -coordinate  $k$ . It is given that the gradient of the line  $CQ$  is  $\frac{4}{5}$ .

- d) Find the value of  $k$ .
- e) **Hence**, calculate the area of triangle  $CQB$ .

a)

$$3y = -x + 18$$

$$y = -\frac{1}{3}x + 6$$

$$A(0, 6)$$

b)

$$\text{when } y = 0$$

$$x = 18$$

$$B(18, 0)$$

c)

$$\text{Length of } AB = \sqrt{(0 - 18)^2 + (6 - 0)^2} = 18.974 = 19.0 \text{ unit}$$

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### Elementary Math Topical (Coordinate Geometry)

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d)

$$C(6, 0)$$

Eqn of  $CQ$

$$0 = \frac{4}{5}(6) + c$$

$$c = -\frac{24}{5}$$

$$y = \frac{4}{5}x - \frac{24}{5}$$

$$y = -\frac{1}{3}x + 6$$

Solve simultaneously

$$\frac{4}{5}x - \frac{24}{5} = -\frac{1}{3}x + 6$$

$$\frac{17}{15}x = \frac{54}{5}$$

$$x = \frac{162}{17}$$

$$y = \frac{48}{17}$$

$$\therefore k = \frac{48}{17}$$

e)

$$\text{Area of triangle} = \frac{1}{2}(12)\left(\frac{48}{17}\right) = \frac{288}{17} \text{ units}^2$$