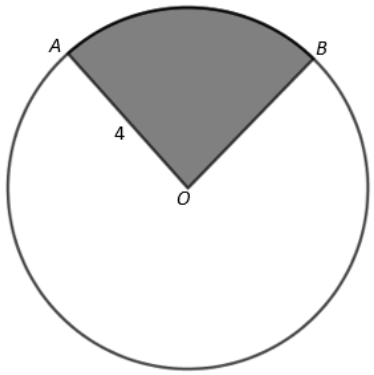
EQUITY
LEARNING PLACE

Elementary Math Topical (Arc Length and Sector Area)

Question 1:

A and *B* lie on the circle, centre *O*, of radius 4 cm. The area of minor sector *AOB* is 12 cm^2.



a) Find, in radians, angle *AOB*.

b) Find an expression, in terms of π, for the length of the major arc *AB*.

a)

$$12 = \frac{1}{2}(4)^2\theta$$

$$\theta = 1.5 \text{ radian}$$

b)

$$\text{Length of major arc} = 4(2\pi - 1.5) = (8\pi - 6) \text{ cm}$$

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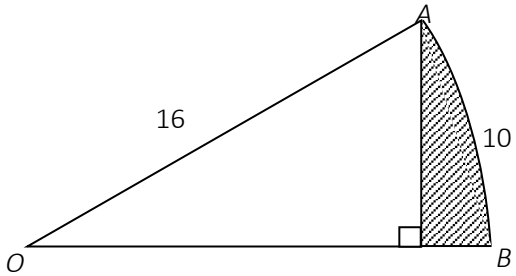
Question 2:

In the diagram, *AOB* is a sector of a circle with centre at *O* and radius 16 cm. *AC* is perpendicular to the radius *OB.*



a) Given that the length of the arc *AB* = 10 cm, show that $\angle AOC = 35.8°$

Find the length of

b) *AC*,

c) *OC*.

d) Calculate the perimeter of the shaded region

e) Calculate the area of the shaded region.

a)

$$\frac{\theta}{360} \times 2\pi(16) = 10$$

$$\theta = 35.80986° = 35.8°$$

b)

$$\sin 35.80986 = \frac{AC}{16}$$

$$AC = 9.36155 = 9.36 \text{ cm}$$

c)

$$\cos 35.80986 = \frac{OC}{16}$$

$$OC = 12.9754 = 13.0 \text{ cm}$$

d)

$$\text{Perimeter} = 9.36155 + (16 - 12.9754) + 10 = 22.4 \text{ cm}$$

e)

$$\text{Shaded Area} = \frac{35.80986}{360} \times \pi \times 16^2 - \frac{1}{2}(12.9754)(9.36155)$$

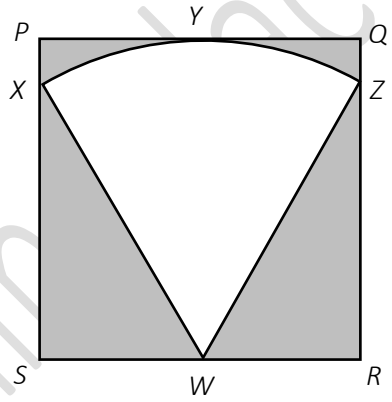
$$= 19.3 \text{ cm}^2$$

Question 3:

PQRS is a square of side 10cm. *W* is the midpoint of *RS*. *WXYZ* is a sector of a circle, centre at *W* and touching *PQ* at *Y*.

a) Explain why the radius of sector *WXYZ* is 10cm.

b) Find the length of arc *XYZ*



a)

$WY = QR$ and WY is the radius of sector $WXYZ$

b)

$$WZ = 10 \text{ cm}, \qquad WR = 5 \text{ cm}$$

$$\cos ZWR = \frac{5}{10}$$

$$\angle ZWR = 60°$$

$$\angle XWZ = 180 - 60 - 60 = 60°$$

$$\text{Arc } XYZ = \frac{60}{360} \times \pi \times (20) = 10.5 \text{ cm}$$

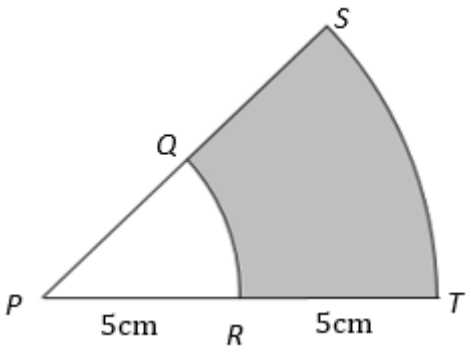
Elementary Math Topical (Arc Length and Sector Area)

Question 4:

The diagram shows the arcs *QR* and *ST* of two circles, centre *P*, with radii 5 cm and 10 cm respectively. PQS and PRT are straight lines. The perimeter of the shaded region is $(5\pi + 10)\text{cm}$

a) Find angle QPR

b) Find the area of the shaded region *QRTS*.



a)

$$(5\pi + 10) = 5 + 5 + 5\theta + 10\theta$$

$$15\theta = 10$$

$$\angle QPR = \theta = 1.5 \text{ radian}$$

b)

$$\text{Shaded area} = 10(1.5) - 5(1.5) = 7.5 \text{ cm}^2$$

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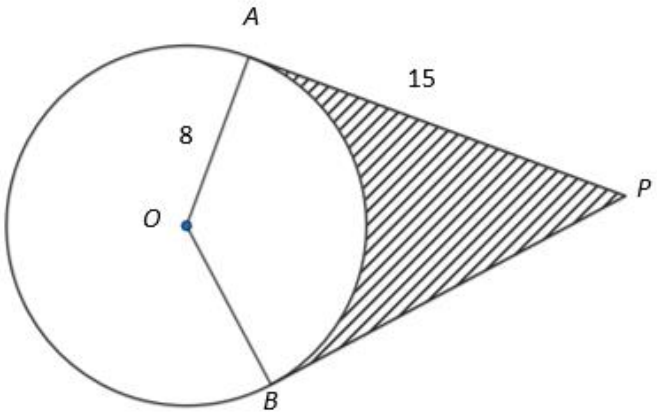
Question 5:



In the diagram, *OA* and *OB* are radii of the circle and *AP* and *BP* are tangents to the circle. Given that *OA* = 8 cm and *AP* = 15 cm.

a) **Stating the appropriate circle properties,** show that angle *AOB* is 2.16 radians,

b) Find the perimeter of the shaded region,

c) Find the area of the shaded region.

a)

$$\angle OAP = 90° \text{ (tangent perpendicular to radius)}$$

$$\tan AOP = \frac{15}{8}$$

$$\angle AOP = 1.080839$$

$$\angle AOB = 2\angle AOP = 2.161678 = 2.16 \text{ radian}$$

(Tangent at external point)

b)

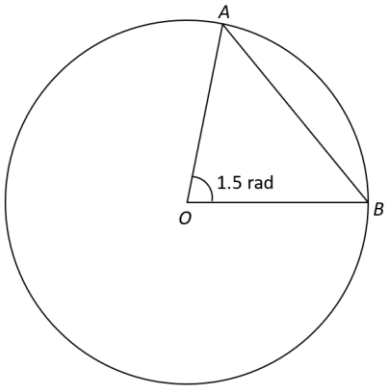
$$\text{Perimeter of shaded region} = 2.161678(8) + 15 + 15 = 47.3 \text{ cm}$$

c)

$$\text{Area of shaded region} = 2 \times \frac{1}{2} \times 8 \times 15 - \frac{1}{2}(8)^2(2.161678) = 50.8 \text{ cm}^2$$

Elementary Math Topical (Arc Length and Sector Area)

Question 6:



The diagram shows a circle, centre *O*. The points *A* and *B* lie on the circumference of the circle. The area of the minor sector *OAB* is 27 cm^2. Calculate the

a) radius of the circle

b) area of triangle *OAB*

c) perimeter of the major sector *OAB*, giving your answer in the form $a+b\pi$

a)

$$27 = \frac{1}{2}r^2(1.5)$$

$$r = 6 \text{ cm}$$

b)

$$\text{Area of triangle} = \frac{1}{2}(6)(6)\sin 1.5 = 18.0 \text{ cm}^2$$

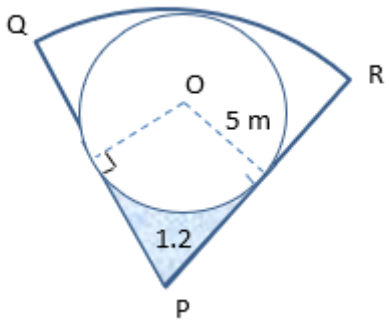
c)

$$\text{Perimeter of major sector} = (2\pi - 1.5)(6) + 12 = 12\pi + 3 \text{ cm}$$

Elementary Math Topical (Arc Length and Sector Area)

Question 7:

The diagram shows a flower bed PQR in the form of a sector of a circle with centre at P and $\angle QPR = 1.2$ rad . A circular plot of land that fits exactly within the boundary of the flower bed is set aside to plant sunflowers. Given that the radius of the circular plot with centre O is 5 m.



a) Prove that the length of PQ is 13.9 m

b) Find the perimeter of the flower bed.

The shaded region of the flower bed is to be fenced up.

c) Calculate the length of fencing required to fence along the perimeter of the shaded region.

a)

$$\sin 0.6 = \frac{5}{OP}$$

$$OP = 8.85516$$

$$\text{Radius of sector} = 8.85516 + 5 = 13.85516 \text{ m}$$

$$PQ = 13.9 \text{ m}$$

b)

$$\text{Perimeter of flower bed} = 2 \times 13.85516 + (13.85516)(1.2) = 44.3 \text{ m}$$

c)

$$\tan 0.6 = \frac{5}{x}$$

$$x = 7.3084797$$

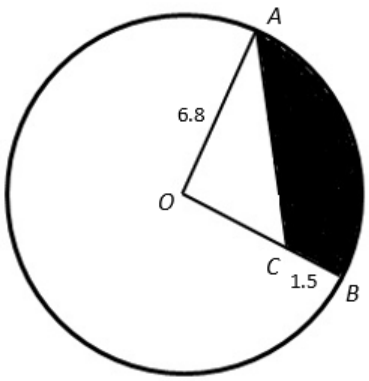
$$\angle O = \pi - 1.2$$

$$\text{Arc length} = (\pi - 1.2)(5) = 5\pi - 6$$

$$\text{Length of fence} = (5\pi - 6) + 7.3084797 \times 2 = 24.3 \text{ cm}$$

Question 8:

In the diagram, *O* is the centre of a circle with radius 6.8 cm. The area of the sector *AOB* is 8π cm^2.



a) Show that angle *AOB* is approximately 1.087 radians.

b) *C* is a point on *OB* such that *BC* is 1.5 cm. Find the area of the shaded region.

a)

$$8\pi = \frac{1}{2}(6.8^2)(\theta)$$

$$\theta = 1.087056 = 1.087 \text{ radian}$$

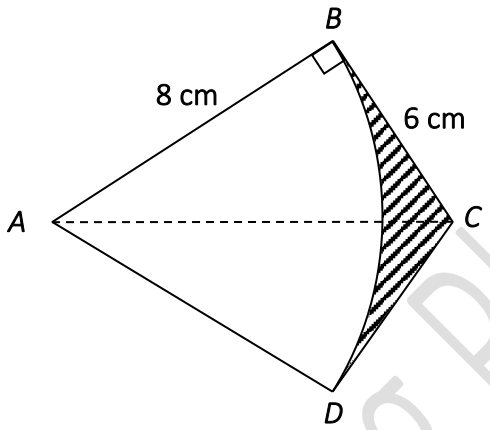
b)

$$\text{Shaded area} = 8\pi - \frac{1}{2}(6.8)(6.8 - 1.5)\sin 1.087056 = 9.18 \text{ cm}^2$$

Elementary Math Topical (Arc Length and Sector Area)

Question 9:

ABCD is a piece of cardboard *ABD* is a sector with centre *A* and angle *ABC* is a right angle. *AC* bisects angle *BAD*. *AB* is 8 cm and *BC* is 6 cm.



a) Show that angle *BAD* is 1.287 radians.

b) Find the perimeter of the shaded region.

c) Find the area of the shaded region.

d) The shaded area is cut away and sector *ABD* is used to form a cone. Find the radius of the base of the cone.

a)

$$\tan BAC = \frac{6}{8}$$

$$\angle BAC = 0.6435011$$

$$\angle BAD = 2\angle BAC = 1.287 \text{ radian}$$

b)

$$\text{Perimeter} = 8(1.28700) + 6 + 6 = 22.3 \text{ cm}$$

c)

$$\text{Shaded area} = 2 \times \frac{1}{2} \times 6 \times 8 - \frac{1}{2}(8)^2(1.28700) = 6.82 \text{ cm}^2$$

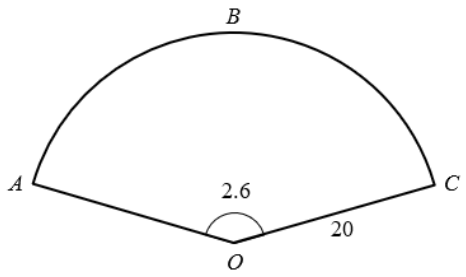
d)

$$8(1.28700) = 2\pi r$$

$$r = 1.64 \text{ cm}$$

Question 10:

OABC is a sector of a circle with centre *O* and radius of 20 cm. $\angle AOC = 2.6$ rad.



a) Calculate the length of the arc *ABC*

b) Sector *OABC* is used to form a conical party hat by joining *OA* and *OC* together. Calculate the base radius of the conical party hat.

c) Convert 2.6 rad to degrees.

a)

$$\text{Arc } ABC = 2.6 \times 20 = 52 \text{ cm}$$

b)

$$52 = 2\pi r$$

$$r = 8.28 \text{ cm}$$

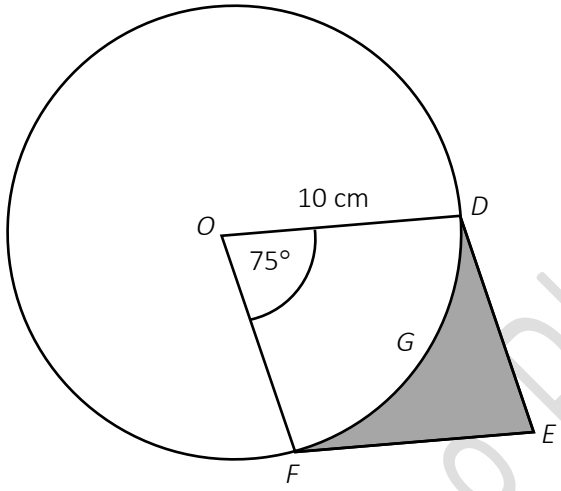
c)

$$2.6 \text{ radian} = \frac{2.6}{\pi} \times 180° = 149.0°$$

Elementary Math Topical (Arc Length and Sector Area)

Question 11:

The diagram below shows the parallelogram *ODEF* and a circle. The circle has centre *O*, radius 10 cm and the angle *FOD* is 75°.



a) Find angle *EOD* in radian measure.

b) Find the length of minor arc *DGF*

c) Find the perimeter of the shaded region

d) Give another special name to parallelogram *ODEF* and explain why.

a)

$$\angle EOD = 37.5° = \frac{37.5}{180} \times \pi = 0.654498 = 0.654 \text{ radian}$$

b)

$$\text{Arc length} = \frac{75}{360} \times 2\pi(10) = 13.089969 = 13.1 \text{ cm}$$

c)

$$\text{Perimeter od shaded} = 13.089969 + 10 + 10 = 33.1 \text{ cm}$$

d) Rhombus

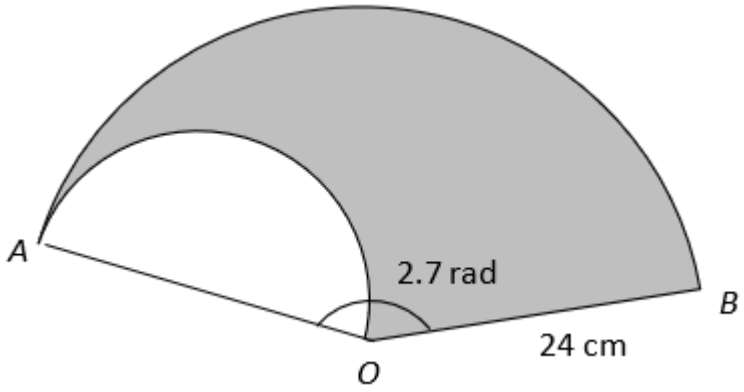
$$OD = OF \text{ (Radius of circle)}$$

Since the adjacent sides are equal the parallelogram is a rhombus.

Elementary Math Topical (Arc Length and Sector Area)

Question 12:

The diagram shows a semi-circle inside a sector *OAB* of radius 24 cm. $\angle AOB = 2.7$ radians.



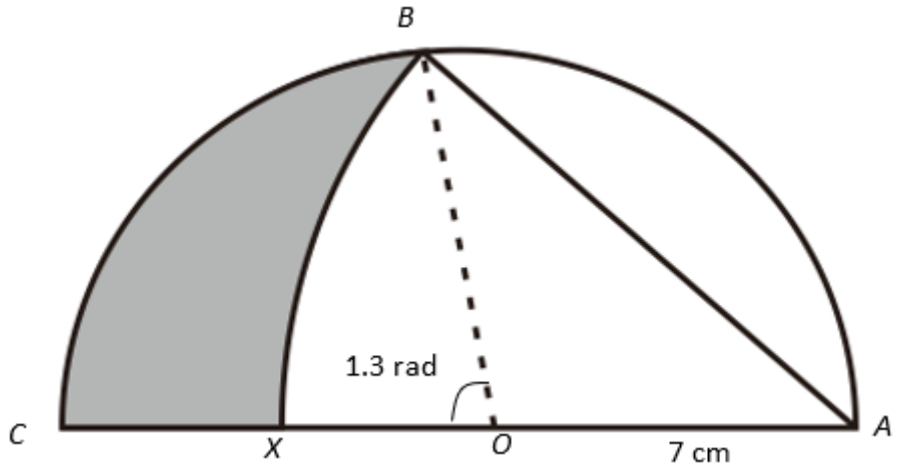
Express the area of the unshaded region as a percentage of the area of the shaded region.

$$\frac{\frac{1}{2} \times \pi \times 12^2}{\frac{1}{2} \times 24^2 \times 2.7} \times 100\% = 29.1\%$$

Elementary Math Topical (Arc Length and Sector Area)

Question 13:

In the diagram, *OABC* is a semicircle of radius 7 cm, with centre *O*. The angle *COB* is 1.3 radians.



a) Show that acute $\angle BAC$ is 0.65 radians.

b) Find the length of the straight line *AB*.

c) *X* is a point on diameter *AC* such that *ABX* is a sector of another circle with centre *A*. Find the arc length *BX*.

d) Find the area of triangle *AOB*.

e) Find the perimeter of the shaded area.

a)

$$\angle BOA = \pi - 1.3$$

$$\angle BAO = \frac{\pi - (\pi - 1.3)}{2} = 0.65 \text{ radian}$$

b)

$$\cos(\pi - 1.3) = \frac{7^2 + 7^2 - AB^2}{2(7)(7)}$$

$$AB = 11.14517 = 11.1 \text{ cm}$$

c)

$$\text{Arc length} = (11.14517)(0.65) = 7.24 \text{ cm}$$

d)

$$\text{Area } AOB = \frac{1}{2}(7)(7)\sin(\pi - 1.3) = 23.6 \text{ cm}^2$$

e)

$$\text{Perimeter of shaded region} = 1.3(7) + 7.24436 = 16.3 \text{ cm}$$

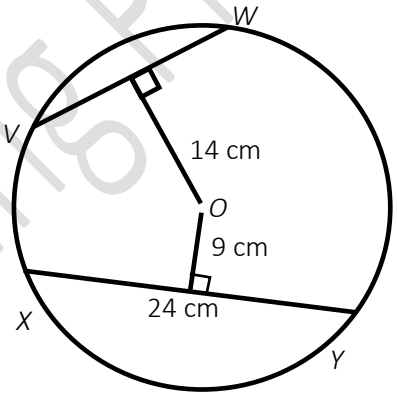
Question 14:

In the diagram shown below, *VW* and *XY* are chords of a circle with centre *O*, where *XY* is 24 cm. The perpendicular distance from *O* to the chord *XY* is 9 cm and the perpendicular distance from *O* to *VW* is 14 cm.

Find

a) the radius of the circle

b) VW



a)

$$OY^2 = 9^2 + 12^2$$

$$OY = 15 \text{ cm} = \text{radius}$$

b)

$$OW = 15 \text{ cm (radius)}$$

$$x^2 + 14^2 = 15^2$$

$$x^2 = 29$$

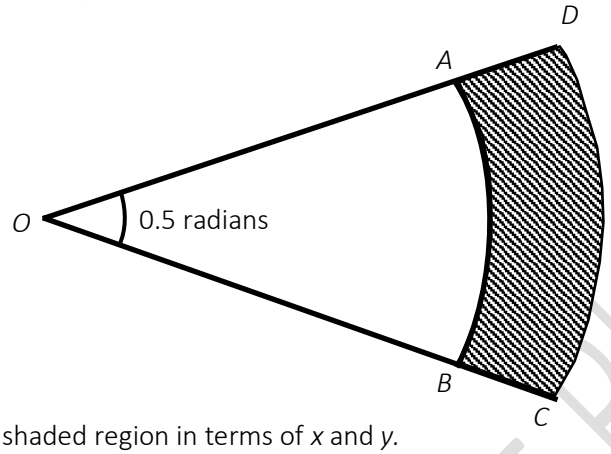
$$x = \sqrt{29}$$

$$VW = 2\sqrt{29} = 10.8 \text{ cm}$$

Elementary Math Topical (Arc Length and Sector Area)

Question 15:

The figure shows arcs, *AB* and *CD*, of two concentric circles with centre at *O*. Their radii, *OB* and *OC*, are *x* cm and *y* cm respectively, and $\angle AOB = 0.5$ radians.



a) Find the area of the shaded region in terms of *x* and *y*.

b) If the perimeter of the shaded region *ABCD* is 120 cm, show that $y = \frac{1}{5}(240 + 3x)$

a)

$$\text{Shaded area} = \frac{1}{2}(y)^2(0.5) - \frac{1}{2}(x)^2(0.5) = \frac{1}{4}(y^2 - x^2)$$

b)

$$120 = 2(y - x) + y(0.5) + x(0.5)$$

$$120 = 2y - 2x + \frac{1}{2}y + \frac{1}{2}x$$

$$120 = \frac{5}{2}y - \frac{3}{2}x$$

$$240 = 5y - 3x$$

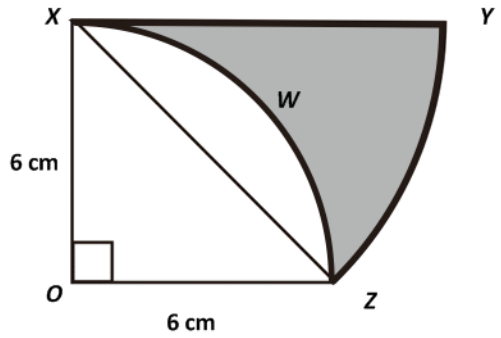
$$5y = 240 + 3x$$

$$y = \frac{1}{5}(240 + 3x)$$

Elementary Math Topical (Arc Length and Sector Area)

Question 16:

In the diagram below, *XOZ* is a right angled triangle, *XY* is parallel to *OZ* and *OX* = *OZ* = 6 cm. *XWZ* is an arc of a circle centre *O* and *YZ* is an arc of circle centre *X*. Find, giving your answer in terms of π, find the area of segment *XWZ*.

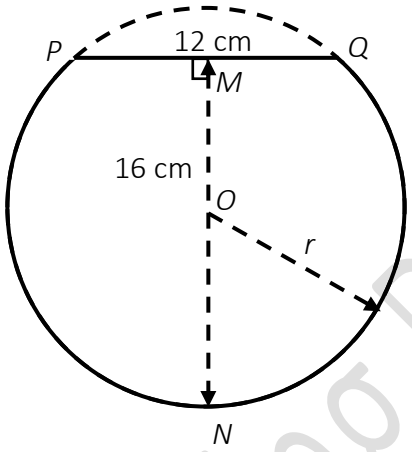


$$\text{Area of segment} = \frac{1}{4}\pi(6)^2 - \frac{1}{2}(6)(6) = 10.3 \text{ cm}^2$$

Elementary Math Topical (Arc Length and Sector Area)

Question 17:

The diagram represents the circular cross-section of a fishbowl. *PQ* represents the horizontal rim of the bowl. *M* is the midpoint of *PQ* and is vertically above *N*. *MN* is perpendicular to *PQ*. *PQ* = 12 cm and *MN* = 16 cm. The circle has a centre *O* and a radius *r* cm.



a) Express *OM* in terms of *r*.

b) Form an equation in *r* and solve it to find the radius of the fishbowl.

a)

$$OM = 16 - r$$

b)

$$OQ = r$$

$$OQ^2 = QM^2 + OM^2$$

$$r^2 = 6^2 + (16 - r)^2$$

$$r^2 = 36 + 256 - 32r + r^2$$

$$32r = 292$$

$$r = 9$$

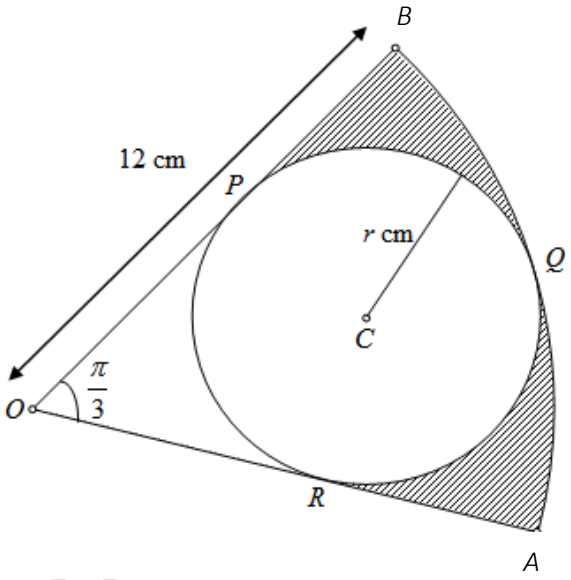
Elementary Math Topical (Arc Length and Sector Area)

Question 18:

In the diagram, the radius of the sector *OAB* is 12 cm and $\angle AOB = \frac{\pi}{3}$. The circle *PQR* with centre *C* and radius *r* cm is inscribed in the sector.

a) Show that *r* = 4.

b) find the area of the shaded region.



a)

$$OQ = 12 \text{ cm}$$

$$OC = 12 - r$$

$$\sin\frac{\pi}{6} = \frac{r}{12 - r}$$

$$\frac{1}{2} = \frac{r}{12 - r}$$

$$12 - r = 2r$$

$$3r = 12$$

$$r = 4$$

b)

$$\text{Area of triangle } OCP = \frac{1}{2}(8)(4)\sin\frac{\pi}{6} = 8 \text{ cm}^2$$

$$\angle PCR = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$\text{reflex } \angle PCR = 2\pi - \frac{5\pi}{6} = \frac{7\pi}{6}$$

$$\text{Area of major sector } PQR = \frac{1}{2}(4)^2\left(\frac{7\pi}{6}\right) = 29.32153 \text{ cm}^2$$

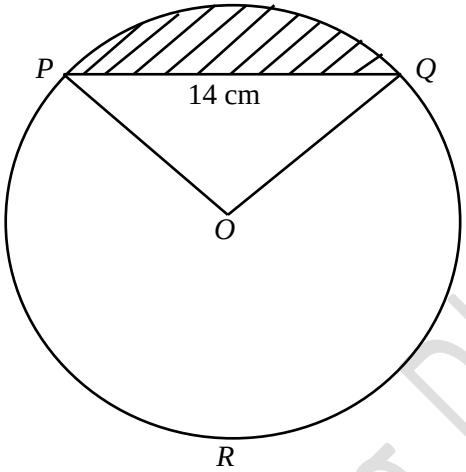
$$\text{Area of sector } OAQB = \frac{1}{2}(12)^2\left(\frac{\pi}{3}\right) = 24\pi$$

$$\text{Shaded area} = 24\pi - 29.32153 - 2 \times 8 = 30.1 \text{ cm}^2$$

Elementary Math Topical (Arc Length and Sector Area)

Question 19:

The figure shows a circle with centre *O*, of radius 10 cm and *PQ* = 14 cm.



a) Show that angle *POQ* = 1.55 radians.

b) Find the length of the major arc *PRQ*.

c) Find the area of the shaded region.

a)

$$\sin\theta = \frac{7}{10}$$

$$\theta = 0.775397$$

$$\angle POQ = 2\theta = 1.55079 = 1.55 \text{ radian}$$

b)

$$\text{Reflex } \angle POQ = 2\pi - 1.55079 = 4.73239$$

$$\text{Major arc length} = 10(4.73239) = 47.3 \text{ cm}$$

c)

$$\text{Shaded area} = \frac{1}{2}(10^2)(1.55079) - \frac{1}{2}(10)^2 \sin 1.55079 = 27.5 \text{ cm}^2$$

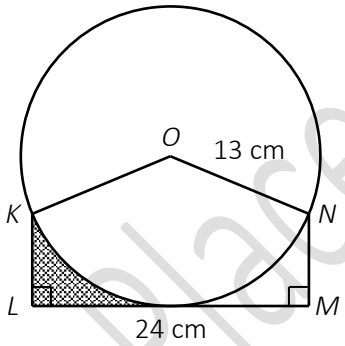
Elementary Math Topical (Arc Length and Sector Area)

Question 20:

In the diagram, the circle with centre *O* passes through *K* and *N*. *LM* is a tangent to the circle with length 24 cm. The radius of the circle is 13 cm.

a) Show that $\angle KON = 134.8°$.

b) Calculate the area of the shaded region.



a)

$$\sin\theta = \frac{12}{13}$$

$$\theta = 67.380135°$$

$$\angle KON = 2\theta = 134.76027 = 134.8°$$

b)

$$\text{Vertical height from } O \text{ to } K = \sqrt{13^2 - 12^2} = 5$$

$$KL = 13 - 5 = 8$$

$$\text{Area of trapezium} = \frac{1}{2}(13 + 8)(12) = 126 \text{ cm}^2$$

$$\text{Area of sector} = \frac{67.380135}{360} \times \pi(13)^2 = 99.3724 \text{ cm}^2$$

$$\text{Shaded area} = 126 - 99.3724 = 26.6 \text{ cm}^2$$