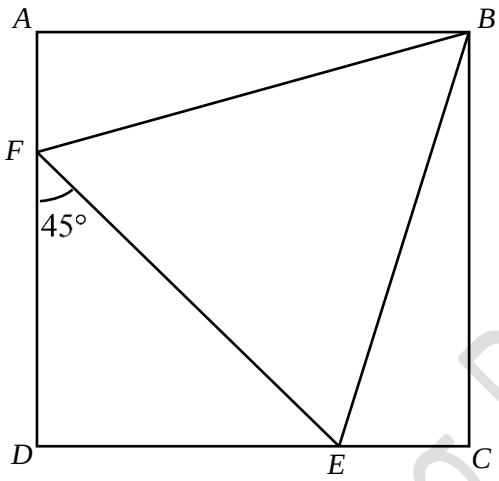
EQUITY
LEARNING PLACE

Elementary Math Topical (Similar and Congruent Triangle)

Question 1

In the diagram, ABCD is a square and angle DFE = $45°$.



a) Prove that *AF* = *EC*.

b) Prove that triangle *ABF* is congruent to triangle *CBE*.

a)

$$\angle FEC = 180 - 90 - 45 = 45° \text{ (sum of angle in triangle)}$$

$$FD = DE$$

$$AF = AD - FD$$

$$AF = DC - DE$$

$$AF = EC$$

b)

$$AB = BC \text{ (side of square)}$$

$$AF = EC \text{ (from part (a))}$$

$$\angle BAF = \angle BCE = 90° \text{ (square)}$$

By SAS, triangle ABF is congruent to triangle CBE

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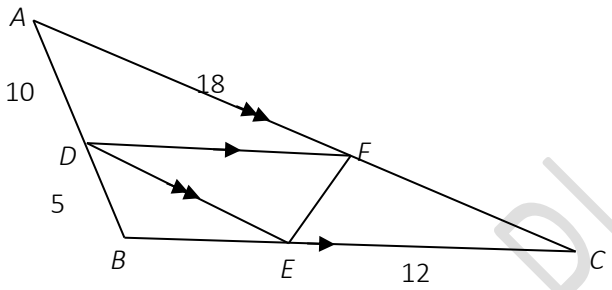
Question 2:

In the diagram, *DF* is parallel to *BC* and *AC* is parallel to *DE*. Given that *DB* = 5 cm,

AD = 10 cm, *EC* = 12 cm and *AF* = 18 cm.



a) State two triangles that are similar to ΔBDE.

b) Prove that ΔCEF is congruent to ΔDFE .

c) Calculate the length of *BC*.

a)

Triangle BAC and Triangle DAF

b)

$DFCE$ is a parallelogram because there are 2 pairs of parallel sides

$\angle ECF = \angle FDE$ (opp angle of parallelogram)

$EC = FD$ (side of parallelogram)

$CF = DE$ (side of parallelogram)

By SAS, triangle CEF is congruent to triangle DFE

c)

$$\frac{BE}{BE + 12} = \frac{5}{15}$$

$$15BE = 5BE + 60$$

$$BE = 6$$

$$BC = 18 \text{ cm}$$

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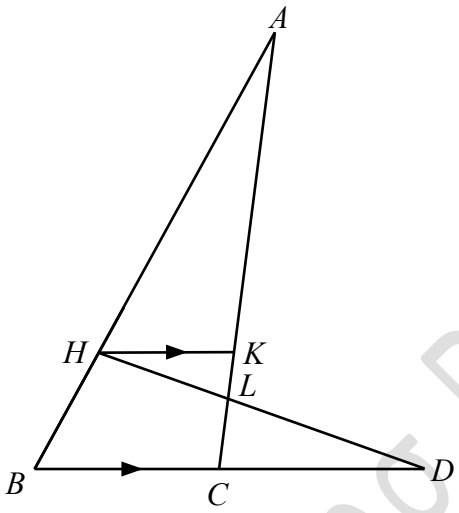
Question 3:

In the diagram below, *AH* = *BD*, *HK* = *HB*, and *HK* is parallel to *BD*.



a) Name the triangle which is congruent to ΔAHK .

b) Hence, prove that ΔAHL is similar to ΔDCL .

c) Given that the *HL*:*CL* = 3:2, find the length of *AH* given that *DC* = 5.5 cm.

a) Triangle DBH

b)

$$\angle ALH = \angle DLC \text{ (vert. opp. angle)}$$

$$\angle HAL = \angle CDL \text{ (part (a) congruent triangle)}$$

By AA, triangle AHL is similar to triangle DCL

c)

$$\frac{HL}{CL} = \frac{AH}{DC}$$

$$\frac{3}{2} = \frac{AH}{5.5}$$

$$AH = 8.25 \text{ cm}$$

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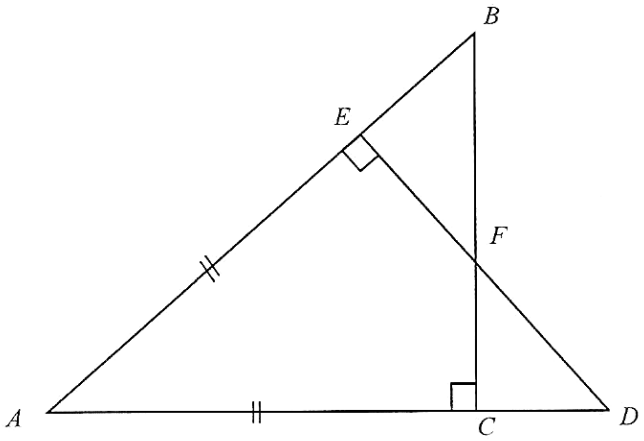
Question 4:

In the diagram, $AE = AC$ and triangles *ACB* and *AED* are right angled triangles. Show that triangles *ACB* and *AED* are congruent.



$\angle CAB = \angle EAD$ (common angle)

$AC = AE$ (given)

$\angle ACB = \angle AED = 90°$

By SAS, Triangle ACB is congruent to Triangle AED

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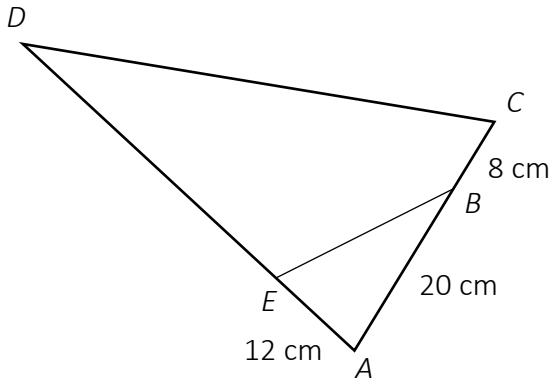
Question 5:

In the diagram, *AE* = 12 cm, *BC* = 8 cm, *AB* = 20 cm and $\angle AEB = \angle ACD$



a) Show that ΔAEB is similar to ΔACD.

b) Find the length of *DE*.

c) Find $\dfrac{Area\ of\ \Delta AEB}{Area\ of\ quadrilateral\ BCDE}$

a)

$\angle AEB = \angle ACD$ (given)

$\angle BAE = \angle DAC$ (common angle)

By AA, Triangle AEB is similar to Triangle ACD

b)

$$\frac{AE}{AC} = \frac{AB}{AD}$$

$$\frac{12}{28} = \frac{20}{AD}$$

$$AD = 46\frac{2}{3}\text{ cm}$$

$$DE = 34\frac{2}{3}\text{ cm}$$

c)

$$\frac{\text{Area of } AEB}{\text{Area of } ACD} = \left(\frac{12}{28}\right)^2 = \frac{9}{49}$$

$$\frac{\text{Area of } AEB}{\text{Area of } BCDE} = \frac{9}{40}$$

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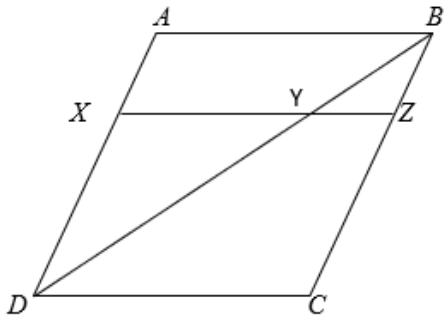
Question 6:

In the figure, *ABCD* is a parallelogram. It is given that *XZ* is parallel to *AB*. *BYD* and *XYZ* are straight lines and $BC = 3BZ$.



a) Identify two triangles similar to ΔBYZ .

b) Calculate $\frac{YD}{YB}$

c) $\frac{\text{area of } \Delta BYZ}{\text{area of } CDYZ}$

a) Triangle *DYX* and Triangle *BDC*

b)

$$\frac{BZ}{BC} = \frac{BY}{BD}$$

$$\frac{1}{3} = \frac{BY}{BD}$$

$$\frac{YD}{YB} = \frac{2}{1} = 2$$

c)

$$\frac{\text{Area of } BYZ}{\text{Area of } BDC} = \frac{1}{9}$$

$$\frac{\text{Area of } BYZ}{\text{Area of } CDYZ} = \frac{1}{8}$$

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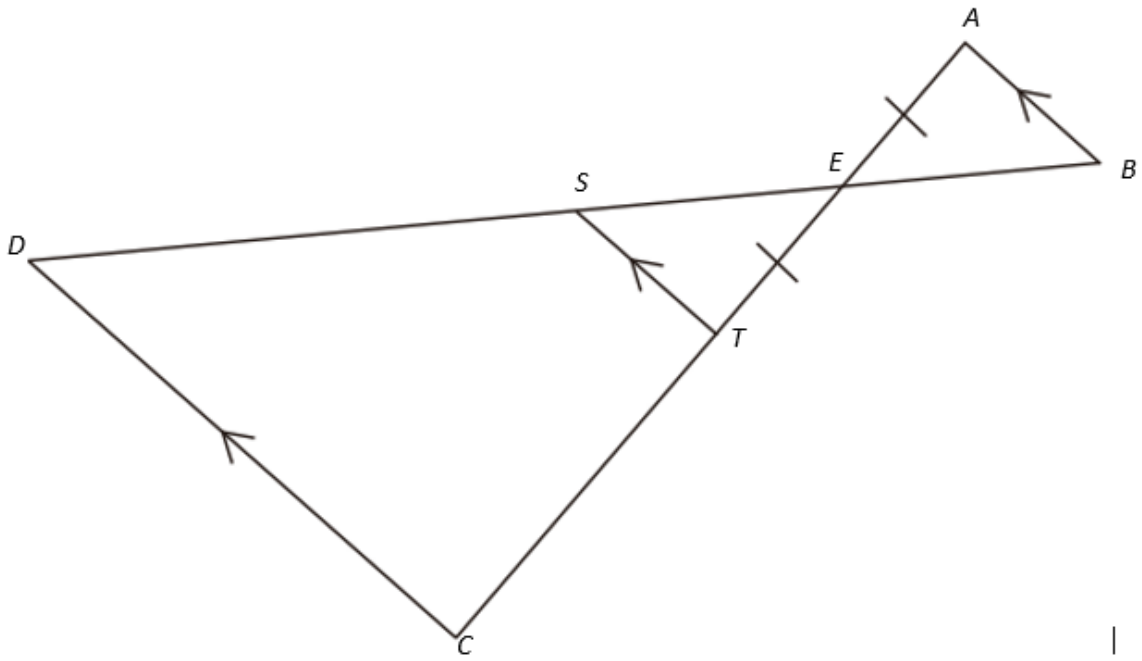
Question 7:

AEC and *BED* are straight lines such that *AE* = *TE* and line *AB* is parallel to the lines *ST* and *DC*.



a) Show that triangles *ABE* and *TSE* are congruent.

The length of *CT* is twice the length of *TE* and *DS* = 23 cm.

b) Calculate the length of *ES*.

c) Find the value of $\frac{Area\ of\ \Delta EST}{Area\ of\ \Delta EDC}$.

d) Given that triangle *ABE* has an area of 20 cm^2. Calculate the area of the quadrilateral *STCD*.

a)

$$\angle AEB = \angle TES \text{ (vert. opp. angle)}$$

$$AE = TE \text{ (given)}$$

$$\angle EAB = \angle ETS \text{ (alt. angle)}$$

By ASA, triangle ABE is congruent to triangle TSE

b)

$$\frac{ES}{ED} = \frac{ET}{EC}$$

$$\frac{ES}{23 + ES} = \frac{1}{3}$$

$$3ES = 23 + ES$$

$$2ES = 23$$

$$ES = 11.5 \text{ cm}$$

c)

$$\frac{\text{Area of } EST}{\text{Area of } EDC} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

d)

$$\text{Area of } ABE = \text{Area of } SET = 20 \text{ cm}^2$$

$$\text{Area of } EDC = 20 \times 9 = 180 \text{ cm}^2$$

$$\text{Area of } STCD = 180 - 20 = 160 \text{ cm}^2$$

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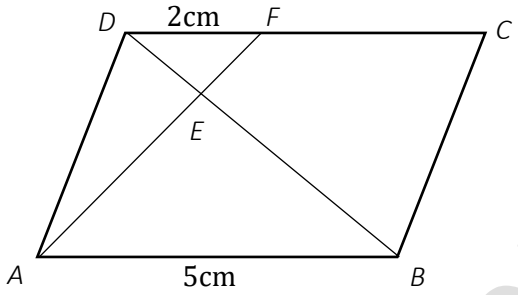
Question 8:

ABCD is a parallelogram and *E* is a point on *BD*. The line *AE* produced meet *CD* at *F*. $DF = 2\text{cm}$ and $AB = 5\text{cm}$.



a) Show that triangles *DEF* and *BEA* are similar.

b) Find the value of $\frac{Area\ of\ \Delta DEF}{Area\ of\ \Delta BEA}$

c) Find the value of $\frac{Area\ of\ \Delta DEF}{Area\ of\ \Delta DEA}$

a)

$$\angle DEF = \angle BEA \text{ (vert. opp. angle)}$$

$$\angle DFE = \angle BAE \text{ (alt angle)}$$

By AA, triangle DEF is similar to triangle BEA

b)

$$\frac{\text{Area of } DEF}{\text{Area of } BEA} = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

c)

$$\frac{\text{Area of } DEF}{\text{Area of } DEA} = \frac{\frac{1}{2} \times EF \times h}{\frac{1}{2} \times EA \times h} = \frac{EF}{EA} = \frac{2}{5}$$

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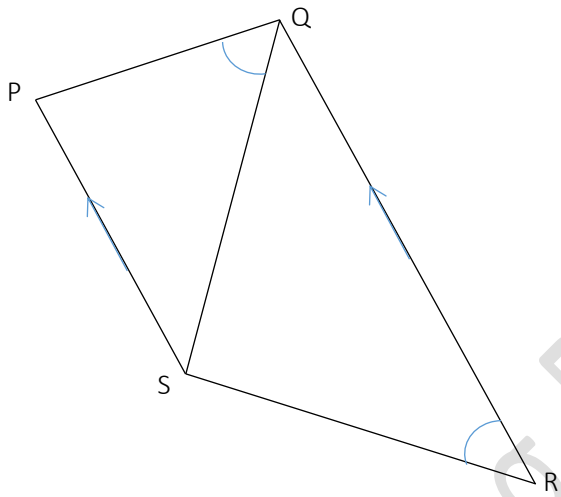
Question 9:

In the diagram, PS is parallel to QR and angle PQS = angle SRQ.



a) Explain why triangles PQS and SRQ are similar.

b) Given that PS = 5 cm, PQ = 6 cm, QS = 8 cm and QR = 13 cm, calculate SR.

a)

$$\angle PQS = \angle SRQ \text{ (given)}$$

$$\angle PSQ = \angle SQR \text{ (alt angle)}$$

By AA, triangle PQS is similar to triangle SRQ

b)

$$\frac{PS}{SQ} = \frac{PQ}{SR}$$

$$\frac{5}{8} = \frac{6}{SR}$$

$$SR = 9.6 \text{ cm}$$

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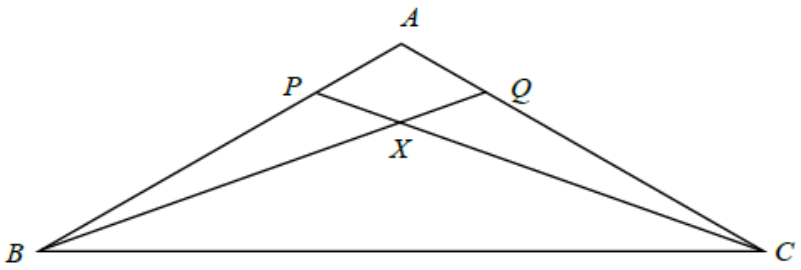
Question 10:

Triangle *ABC* is an isosceles triangle with *AB* = *AC*. *P* and *Q* are points on the line *AB* and *AC* respectively such that *PA* = *QA*. The ratio of *AQ* : *AC* is 1 : 4. The lines *BQ* and *CP* intersect at the point *X*.



a) Show that triangle *BXC* is an isosceles triangle.

b) Given that the area of triangle *ABC* = 45 cm^2, find the area of triangle *BCQ*.

a)

$$\angle BAQ = \angle CAP \text{ (common angle)}$$

$$AP = AQ \text{ (given)}$$

$$AB = AC \text{ (given)}$$

$$\text{By SAS, triangle } ABQ \text{ is congruent to triangle } ACP$$

$$\angle ABQ = \angle ACP$$

$$\text{Since } AB = AC, \angle ABC = \angle ACB$$

$$\angle XBC = \angle ABC - \angle ABQ$$

$$= \angle ACB - \angle ACP$$

$$= \angle XCB$$

$$\therefore \text{ triangle } BXC \text{ is an isosceles triangle.}$$

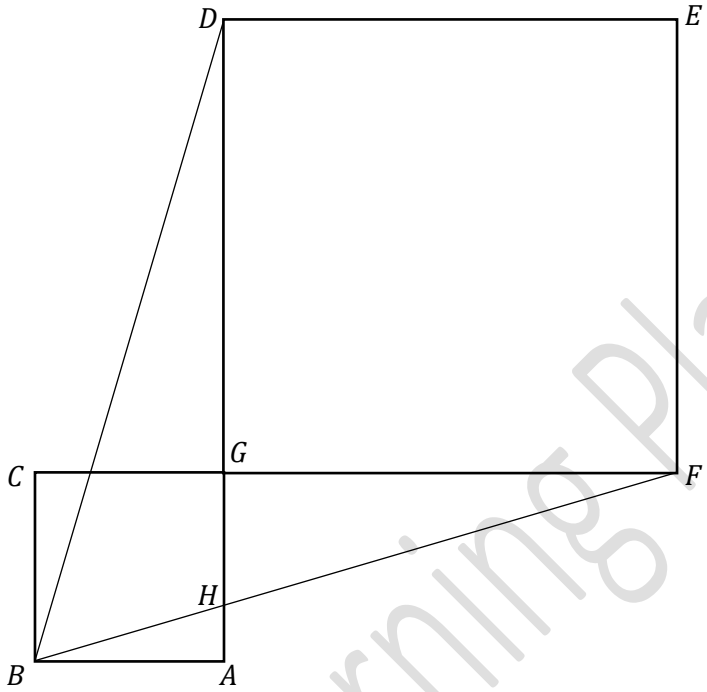
b)

$$\frac{\text{Area of } ABC}{\text{Area of } BCQ} = \frac{\frac{1}{2} \times AC \times h}{\frac{1}{2} \times QC \times h} = \frac{AC}{QC} = \frac{4}{3}$$

$$\text{Area of } BCQ = \frac{3}{4} \times 45 = 33.75 \text{ cm}^2$$

Question 11:

In the diagram, $ABCG$ and $DEFG$ are squares and $\angle AGF = 90°$



a) Prove that triangle BCF is congruent to triangle BAD.

b) Hence, prove that triangle FGH is similar to triangle DAB

a)

$BC = BA$ (side of square)

$\angle BCF = \angle BAD = 90°$ (square)

$CF = AD$ (side of squares)

By SAS, triangle BCF is congruent to triangle BAD

b)

$\angle FGH = \angle DAB = 90°$ (square)

$\angle GFH = \angle ADB$ (congruent triangle from (a))

By AA, triangle FGH is similar to triangle DAB

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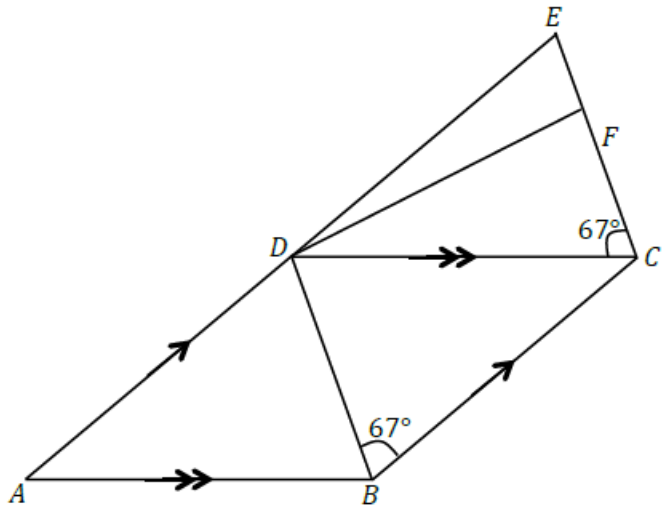
Question 12:

In the diagram below, $ABCD$ is a rhombus. AD is produced to point E and EFC is a straight line. It is given that $\angle DBC = \angle DCE = 67°$ and $EC = 3EF$.



a) Name one triangle which is congruent to triangle EDC, Explain your reasons clearly.

Find the numerical value of

b) $\frac{\text{Area of triangle } EDF}{\text{Area of triangle } EDC}$

c) $\frac{\text{Area of triangle } DFC}{\text{Area of quadrilateral } ABCD}$

a) Triangle *DCB*

$$\angle ECD = \angle DBC = 67°$$

$$\angle EDC = \angle DCB \text{ (alt angle)}$$

$$DC = CB \text{ (side of rhombus)}$$

By ASA, triangle DCB is congruent to triangle EDC

b)

$$\frac{\text{Area of } EDF}{\text{Area of } EDC} = \frac{\frac{1}{2} \times EF \times h}{\frac{1}{2} \times EC \times h} = \frac{EF}{EC} = \frac{1}{3}$$

c)

$$\text{Area of } ABCD = 2 \times \text{Area of EDC}$$

$$\frac{(\text{Area of DFC})}{\text{Area of } ABCD} = \frac{2}{6} = \frac{1}{3}$$