EQUITY
LEARNING PLACE

Elementary Math Topical (Trigonometry)

Question 1:

The area of triangle XYZ is $124.9\,\text{cm}^2$. $XY = 14.6$ cm and $XZ = 19.2$ cm. Find the two possible sizes of angle YXZ.

$$\text{Area of triangle} = \frac{1}{2}ab\sin C$$

$$124.9 = \frac{1}{2}(14.6)(19.2)\sin C$$

$$\sin C = 0.891124$$

$$C = \sin^{-1}(0.891124)$$

$$C = 63.01488 \text{ or } 180 - 63.01488$$

$$C = 63.0° \text{ or } 117.0°$$

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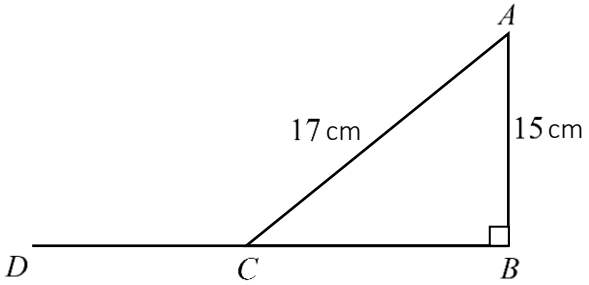
Question 2:

In the diagram, $\angle ABC = 90°$, $AB = 15$ cm, $AC = 17$ cm and BCD is a straight line.



a) Find the length of BC.

Express as a fraction in its simplest form, the value of

b) $\tan \angle BAC$

c) $\cos \angle ACD$

a) By Pythagoras' Theorem,

$$BC^2 + AB^2 = AC^2$$

$$BC^2 = 17^2 - 15^2$$

$$BC = 8$$

b)

$$\tan \angle BAC = \frac{8}{15}$$

c)

$$\cos \angle ACD = -\frac{8}{17}$$

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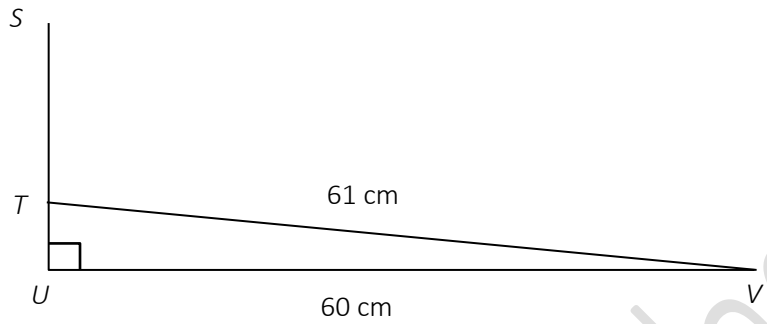
Question 3:

In the diagram below, angle *TUV* = 90°, *UV* = 60 cm, *TV* = 61 cm and *STU* is a straight line.



Find, as a fraction in its lowest term,

a) $\sin T\hat{V}U$

b) $\cos S\hat{T}V$

a)

$$TU^2 = 61^2 - 60^2$$

$$TU = 11$$

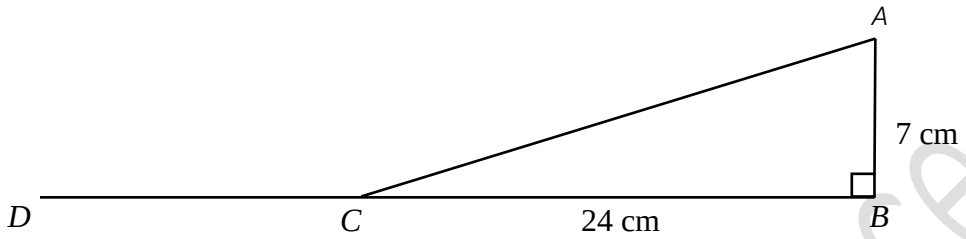
$$\sin \angle TVU = \frac{11}{61}$$

b)

$$\cos \angle STV = -\frac{11}{61}$$

Question 4:

In the diagram, *BCD* is a straight line. *AB* = 7 cm, *BC* = 24 cm and angle *ABC* = 90^{o}.



a) $\sin \angle ACB,$

b) $\cos \angle ACD$

a)

$$AC^2 = 24^2 + 7^2$$

$$AC = 25$$

$$\sin \angle ACB = \frac{7}{25}$$

b)

$$cos \angle ACD = -\frac{24}{25}$$

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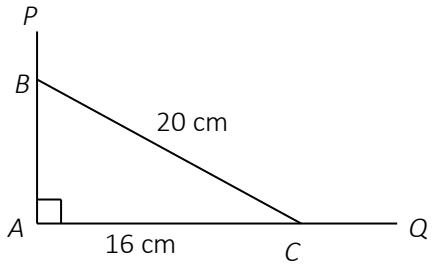
Question 5:

ABC is a triangle in which $\angle BAC$ is $90°$, *AC* = 16 cm and *BC* = 20 cm. *AB* is produced to *P* and *AC* is produced to *Q*.



a) Calculate $\angle ABC$

Express as a fraction

b) sin $\angle BCA$

c) cos $\angle BCQ$.

a)

$$\sin \angle ABC = \frac{16}{20}$$

$$\angle ABC = \sin^{-1}\left(\frac{16}{20}\right) = 53.1°$$

b)

$$AB^2 = 20^2 - 16^2$$

$$AB = 12$$

$$\sin \angle BCA = \frac{12}{20} = \frac{3}{5}$$

c)

$$\cos \angle BCQ = -\frac{16}{20} = -\frac{4}{5}$$

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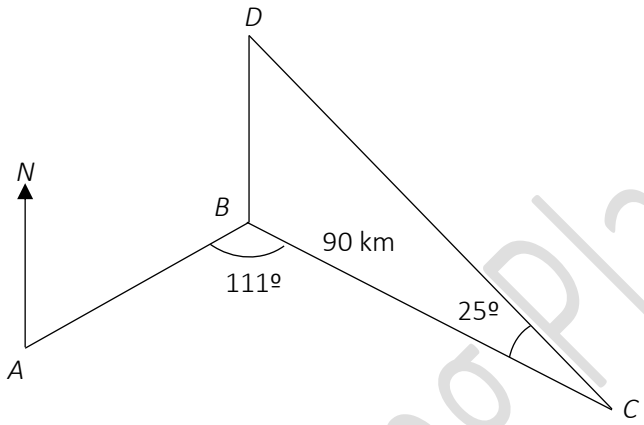
Question 6:

A ship sails from a Port *A* at 10 00 on a bearing of 050° towards port *B*. It sails at an average speed of 30 km/h, reaching Port *B* at 12 35. From port *B*, it sails to Port *C* which is 90 km away. The ship sets sail again to Port *D*, which is due north of Port *B*. The figure below shows the route taken by the ship.



Given also that $\angle ABC = 111°$ and $\angle BCD = 25°$.

a) Show that the distance of *AB* = 77.5 km.

b) Calculate the distance of *AC*.

c) Calculate the distance of *BD*

d) Find the bearing of *A* from *C*, to the nearest degree.

e) The ship sent a distress signal that a crew member had fallen overboard. A search and rescue team decided to comb the area enclosed by port *B, C* and *D*. Find this area.

f) A helicopter, *H* is hovering at a point 3 km vertically above port *B*. Calculate the angle of elevation of the helicopter when viewed from Port C.

a)

$$\text{Distance} = 30 \times 2\frac{35}{60} = 77.5 \text{ km}$$

b)

$$\cos 111 = \frac{77.5^2 + 90^2 - AC^2}{2(77.5)(90)}$$

$$AC = 138.222$$

$$AC = 138 \text{ km}$$

c)

$$\angle DBA = 180° - 50° = 130°$$

$$\angle DBC = 360° - 130° - 110° = 120°$$

$$\angle BDC = 180° - 120° - 25° = 35°$$

$$\frac{BD}{\sin 25} = \frac{90}{\sin 35}$$

$$BD = 66.3131$$

$$BD = 66.3 \text{ km}$$

d)

$$\frac{\sin BAC}{90} = \frac{\sin 111}{138.222}$$

$$\angle BAC = 37.436°$$

$$N_A AC = 50 + 37.436 = 87.436$$

$$N_c CA = 180 - 87.436 = 92.564$$

$$\text{Bearing of } A \text{ from } C = 360 - 92.564 = 267.436 = 267°$$

e)

$$\text{Area} = \frac{1}{2}(66.3131)(90)\sin 120 = 2584.297 = 2580 \text{ km}$$

f)

$$\tan\theta = \frac{3}{90}$$

$$\theta = 1.909$$

$$\therefore \text{ angle of elevation is } 1.9°$$

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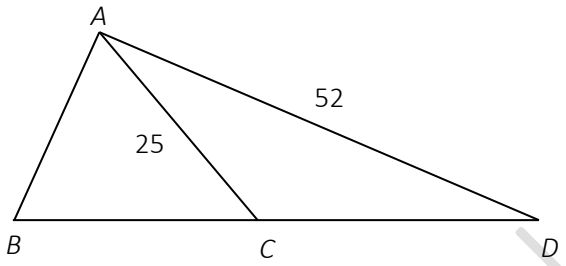
Question 7:

In the figure below, *BCD* is a straight line. *AD* = 52 cm, *AC* = 25 cm and $\sin\angle ADC = \frac{5}{13}$ Calculate the value of $\sin\angle ACB$, giving your answer as a fraction in its lowest terms.



$$\frac{\sin ACB}{52} = \frac{\sin ADC}{25}$$

$$\sin ACB = \frac{\frac{5}{13}}{25} \times 52 = \frac{4}{5}$$

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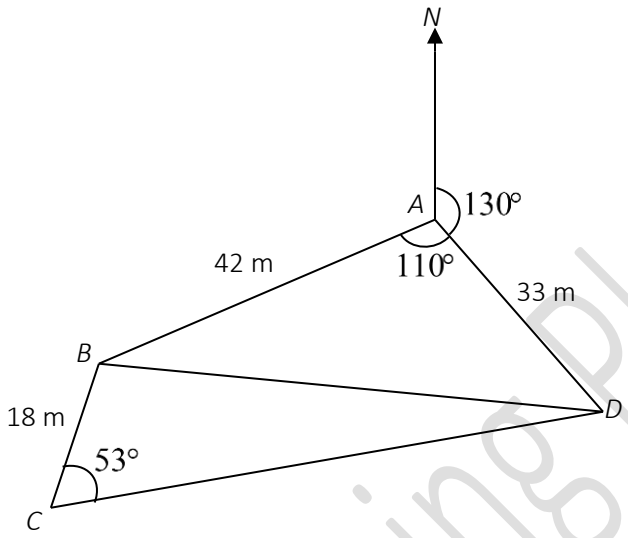
Question 8:

A, *B*, *C* and *D* are points on horizontal ground. *D* is 33 m from *A* on a bearing of $130°$.
$\angle BAD = 110°$, $\angle BCD = 53°$, *BC* = 18 m and *AB* = 42 m.



Calculate

a) the length of *BD*,

b) $\angle BDC$

c) the area of triangle *ABD*,

d) the shortest distance from *A* to *BD*

e) the bearing of *A* from *B*

f) A tower of height 65 m is erected vertically at point *A*. Find the greatest angle of elevation when viewed from along the path *BD*.

a)

$$\cos 110 = \frac{42^2 + 33^2 - BD^2}{2(42)(33)}$$

$$BD = 61.65289$$

$$BD = 61.7 \text{ m}$$

b)

$$\frac{\sin BDC}{18} = \frac{\sin 53}{61.65289}$$

$$\angle BDC = 13.483617 = 13.5°$$

c)

$$\text{Area} = \frac{1}{2}(42)(33)\sin 110 = 651.206 = 651\ \text{m}^2$$

d)

$$\frac{1}{2}(61.65289)(h) = 651.206$$

$$h = 21.1249 = 21.1\ \text{m}$$

e)

$$\angle N_A AB = 360 - 110 - 130 = 120$$

$$\text{Bearing of } A \text{ from } B = 180 - 120 = 060°$$

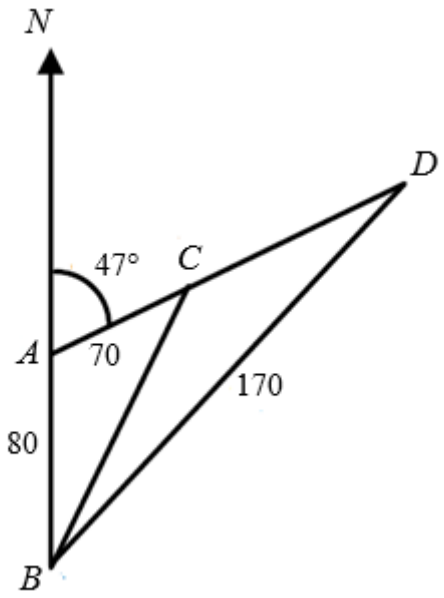
f)

$$\tan\theta = \frac{65}{21.1249}$$

$$\theta = 71.99588 = 72.0°$$

Question 9:

The diagram shows four points, *A*, *B*, *C* and *D* on the ground. The bearing of *D* from *A* is 047°.
B is 80 m due south of *A* and *C* is on the line *AD*, such that *C* is 70 m away from *A* and *BD* is 170 m apart.



Find

a) the distance *BC,*

b) $\angle ADB$,

c) the bearing of *D* from *B.*

d) There is a tree planted at *A*. A boy measures the angle of elevation of the top of the tree from *B* and finds that it is 23°. Find the height of the tree, assuming that the boy's height is negligible.

e) The boy decides to walk from *B* to *D* along the line *BD*, until he reaches a point *X* such that *AX* is the shortest. Find the distance *AX*.

a)

$$\angle BAC = 180 - 47 = 133°$$

$$\cos 133 = \frac{80^2 + 70^2 - BC^2}{2(80)(70)}$$

$$BC = 137.61679 = 138 \text{ m}$$

b)

$$\frac{\sin ADB}{80} = \frac{\sin 133}{170}$$

$$\angle ADB = 20.1309 = 20.1°$$

c)

$$\text{Bearing of } D \text{ from } B = 180 - 20.1309 - 133 = 26.869 = 026.9$$

d)

$$\tan 23 = \frac{h}{80}$$

$$h = 33.95798 = 34.0 \text{ m}$$

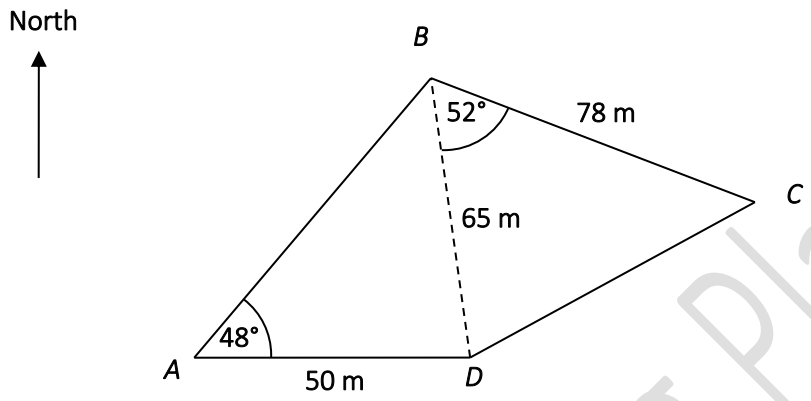
e)

$$\sin 26.869 = \frac{AX}{80}$$

$$AX = 36.156 = 36.2 \text{ m}$$

Question 10:

The diagram shows a field *ABCD* on horizontal ground, with a path *BD* cutting through the field. *D* is due east of *A*. Angle *BAD* = 48° and angle *CBD* = 52°. *AD* = 50 m. *BD* = 65 m. *BC* = 78 m.



Calculate

a) the bearing of *A* from *B*,

b) angle *ABD*

c) area of triangle *CBD*

d) *CD*

e) There is a tower of height 20 m at *C*. A boy walks from *B* to *D* and at a point between *B* and *D*, he stopped and the angle of elevation to the top of the tower is the largest. Find the angle of elevation to the top of the tower.

a)

$$\angle N_A AB = 90 - 48 = 42°$$

$$\angle N_B BA = 180 - 42 = 138°$$

$$\text{Bearing of } A \text{ from } B = 360 - 138 = 222°$$

b)

$$\frac{\sin ABD}{50} = \frac{\sin 48}{65}$$

$$\angle ABD = 34.865356 = 34.9°$$

c)

$$\text{Area } CBD = \frac{1}{2}(78)(65)\sin 52 = 1997.607 = 2000 \text{ m}^2$$

d)

$$\cos 52 = \frac{78^2 + 65^2 - CD^2}{2(78)(65)}$$

$$CD = 63.7667 = 63.8 \text{ m}$$

e) Let x be shortest distance from C to BD

$$\sin 52 = \frac{x}{78}$$

$$x = 61.4648$$

Let θ be angle of elevation

$$\tan\theta = \frac{20}{61.4648}$$

$$\theta = 18.0°$$

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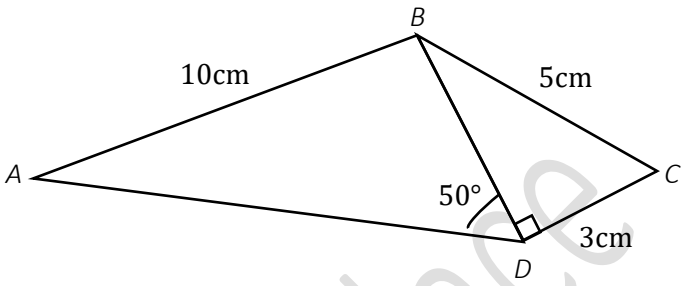
Question 11:

In the diagram, angle $BDA = 50°$, angle $BDC = 90°, AB = 10\text{cm}, BC = 5\text{cm}$ and $CD = 3\text{cm}$.

a) Find BD

b) Find angle BAD



a)

$$\text{BD}^2 = 5^2 - 3^2$$

$$\text{BD} = 4$$

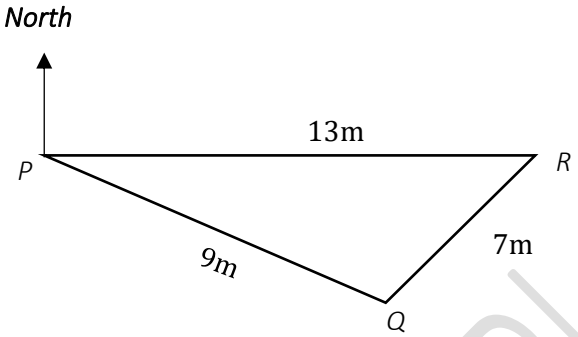
b)

$$\tan BAD = \frac{4}{10}$$

$$\angle BAD = 21.8°$$

Question 12:

P, *Q* and *R* are 3 locations of a park such that *R* is due east of *P*. The dimensions of the park are given as $PQ = 9\text{m}$, $PR = 13\text{m}$ and $QR = 7\text{m}$.



a) Calculate angle *QPR*.

b) Show that obtuse angle $PQR = 108.0°$, correct to nearest 1 decimal place.

c) Find the bearing of *R* from *Q*.

d) Find the bearing of *P* from *Q*.

There is a tree of height 2.02m at *Q*. Calculate

e) the shortest distance from *Q* to *PR*

f) the greatest angle of elevation of the top of the tree from a point along the line *PR*.

a)

$$\cos QPR = \frac{13^2 + 9^2 - 7^2}{2(13)(9)}$$

$$\angle QPR = 30.79838 = 30.8°$$

b)

$$\cos PQR = \frac{9^2 + 7^2 - 13^2}{2(9)(7)}$$

$$\angle PQR = 108.03050 = 108.0°$$

c)

$$\angle PRQ = 180 - 108.03050 - 30.79838 = 41.17112°$$

$$\text{Bearing of } R \text{ from } Q = 180 - 41.17112 - 90 = 48.82888 = 048.8°$$

d)

$$N_Q QP = 108.03050 - 48.82888 = 59.2017°$$

$$\text{Bearing of } P \text{ from } Q = 360 - 59.2017 = 300.8°$$

e) Let x be shortest distance

$$\sin 30.79838 = \frac{x}{7}$$

$$x = 3.58413$$

$$x = 3.58 \text{ m}$$

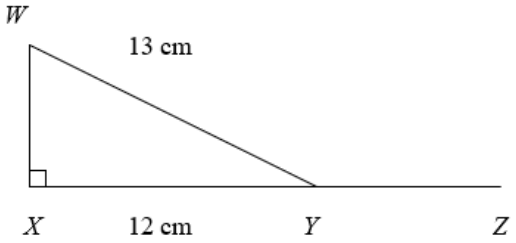
f) Let angle of elevation be θ

$$\tan\theta = \frac{2.02}{3.58413}$$

$$\theta = 29.4°$$

Question 13:

In triangle *WXY*, angle *WXY* = 90° and *XY* is produced to *Z*.



Write down the value of

a) $\tan\angle WYX$

b) $\cos\angle WYZ$

c) Calculate the perpendicular distance from *X* to *WY*.

a)

$$WX^2 = 13^2 - 12^2$$

$$WX = 5$$

$$\tan\angle WYX = \frac{5}{12}$$

b)

$$\cos\angle WYZ = -\frac{12}{13}$$

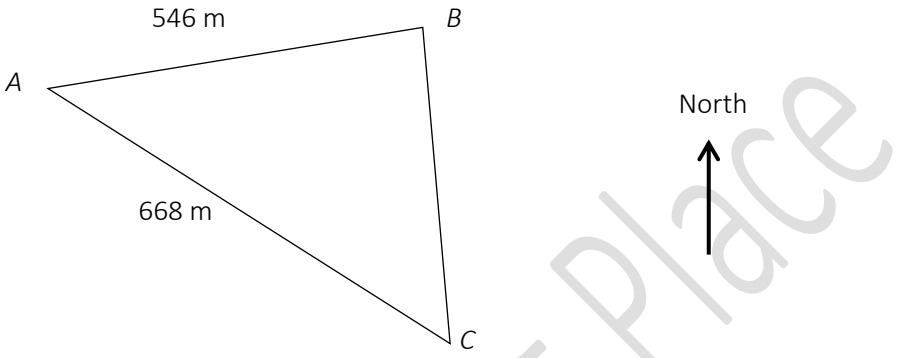
c) Let the perpendicular distance be x

$$\frac{1}{2}(x)(13) = \frac{1}{2}(5)(12)$$

$$x = \frac{60}{13}\text{ cm}$$

Question 14:

Town *A*, *B* and *C* are situated on an island. The bearing of *B* from *A* is $073°$ and the bearing of *C* from *A* is $104°$. The distance between *A* and *B* is 546 m and the distance between *A* and *C* is 668 m.



Calculate

a) angle BAC

b) the distance *BC*

c) angle ACB

Town *B* has a lighthouse. The angle of elevation to the top of the lighthouse from *A* is 10°. Calculate

d) the height of the lighthouse

e) the greatest possible angle of depression from the top of the lighthouse to a point between Town *A* and Town *C*.

a)

$$\angle BAC = 104 - 73 = 31°$$

b)

$$\cos 31 = \frac{546^2 + 668^2 - BC^2}{(2)(546)(668)}$$

$$BC = 345.0712 = 345 \text{ m}$$

c)

$$\frac{\sin ACB}{546} = \frac{\sin 31}{345.0712}$$

$$\angle ACB = 54.5809 = 54.6°$$

d) Let h be height of lighthouse

$$\tan 10 = \frac{h}{546}$$

$$h = 96.2753 = 96.3 \text{ m}$$

e) Let x be shortest distance from B to AC

$$\sin 31 = \frac{x}{546}$$

$$x = 281.2108$$

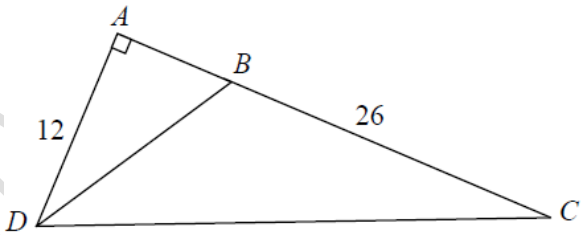
Let θ be largest angle of elevation

$$\tan \theta = \frac{10}{281.2108}$$

$$\theta = 2.0°$$

Question 15:

In the diagram, *ABC* is a straight line and $\angle DAB = 90°$. *AD* = 12 cm, *BC* = 26 cm and $\sin \angle ABD = \frac{4}{5}$. Find, as a fraction, the value of $\cos \angle DBC + \tan \angle ADC$.



$$\sin ABD = \frac{12}{DB}$$

$$\frac{4}{5} = \frac{12}{DB}$$

$$DB = 15$$

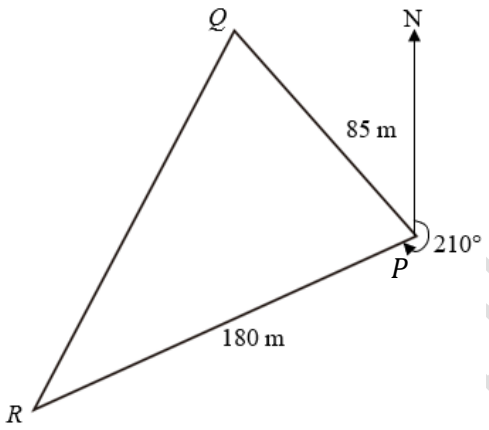
$$AB^2 = 15^2 - 12^2$$

$$AB = 9$$

$$\cos DBC + \tan ADC = -\frac{9}{15} + \frac{35}{12} = \frac{139}{60}$$

Question 16:

In the diagram, *PQR* represents a horizontal field such that *PQ* = 85 m and *R* is 180 away In the diagram, *PQR* represents a horizontal field such that *PQ* = 85 m and *R* is 180 away



a) Find the bearing of *P* from *R*

Given that $\angle PQR$ is 70°, calculate

b) $\angle QRP$

c) area of the field

d) the shortest distance from *Q* to *PR*.

e) A drone is flying above point *Q* at a height of 450 m. Find the greatest possible angle of depression from the drone to a point along the path *PR*.

a)

$$\angle N_P PR = 360 - 210 = 150°$$

$$\text{Bearing of } P \text{ from } R = 180 - 150 = 30°$$

b)

$$\frac{\sin QRP}{85} = \frac{\sin 70}{180}$$

$$\angle QRP = 26.34299 = 26.3°$$

c)

$$\angle QPR = 180 - 70 - 26.34299 = 83.657$$

$$\text{Area} = \frac{1}{2}(85)(180)\sin 83.657 = 7603.169 = 7600 \text{ m}^2$$

d) Let x be the shortest distance from Q to PR

$$\sin 83.657 = \frac{x}{85}$$

$$x = 84.4797 = 84.5 \text{ m}$$

e) Let θ be the greatest angle of depression

$$\tan\theta = \frac{450}{84.4797}$$

$$\theta = 79.4°$$

Question 17:

The sine of an angle is 0.4321.

Give two possible values for the angle.

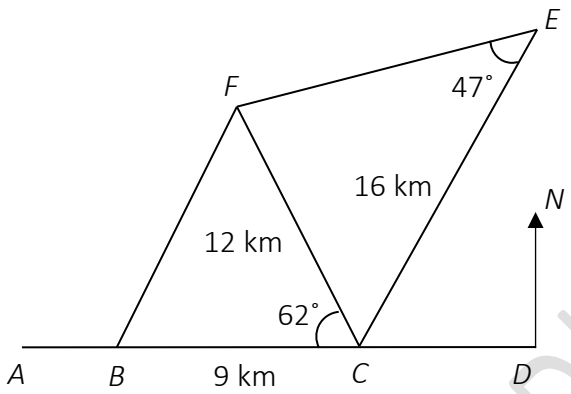
$$\sin x = 0.4321$$

$$x = 25.6° \text{ or } 154.4°$$

Elementary Math Topical (Trigonometry)

Question 18:

Points *A*, *B*, *C*, *D*, *E* and *F* are on horizontal ground. *A, B* and *C* are due west of *D*. *E* is due north of *D*.



Given that *BC* = 9 km, *CF* = 12 km, *CE* = 16 km, $\angle BCF = 62°$ and $\angle CEF = 47°$,

a) calculate the length of *BF*,

b) calculate the acute angle *CFE*,

c) find the area of the quadrilateral *BCEF*,

d) find the bearing of *E* from *C*.

A vertical flag is 3490 m vertically above *F*.

e) Find the angle of elevation of the top of the flag from *C*.

a)

$$\cos 62 = \frac{9^2 + 12^2 - BF^2}{2(9)(12)}$$

$$BF = 11.11729 = 11.1 \text{ km}$$

b)

$$\frac{\sin CFE}{16} = \frac{\sin 47}{12}$$

$$\angle CFE = 77.1971 = 77.2°$$

c)

$$\angle FCE = 180 - 77.1971 - 47 = 55.802866$$

$$\text{Area of quadrilateral} = \frac{1}{2}(12)(9)\sin(62) + \frac{1}{2}(12)(16)\sin 55.802866$$

$$\text{Area of quadrilateral} = 127.816 = 128 \text{ m}^2$$

d)

$$\text{Bearing of } E \text{ from } C = 55.802866 + 62 - 90 = 027.8°$$

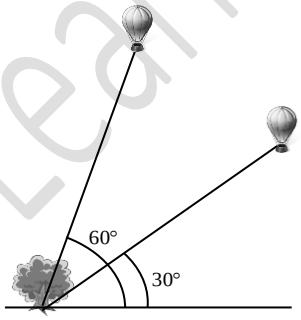
e) Let angle of elevation be θ

$$\tan\theta = \frac{3490}{12000}$$

$$\theta = 16.2°$$

Question 19:

A hot air balloon was tied with a 400 m rope to the bottom of a tree. At first, the angle between the rope and ground was $60°$ and finally decreased to $30°$. Find the vertical distance that the hot air balloon descended.



Let original vertical distance be x

$$\sin 60 = \frac{x}{400}$$

$$x = 400\sin 60$$

Let new vertical distance be y

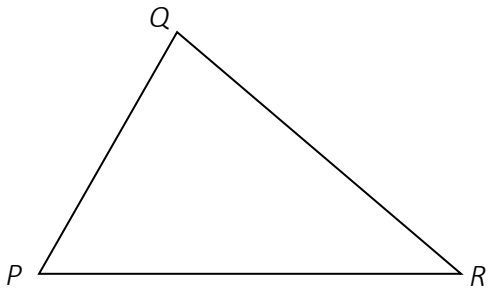
$$\sin 30 = \frac{y}{400}$$

$$y = 400\sin 30$$

$$\text{Change in vertical distance} = 400\sin 60 - 400\sin 30 = 146 \text{ m}$$

Question 20:

In the diagram, the ratio of $PQ : QR : PR = 5 : 6 : 7$.



Calculate

a) the value of $\cos \angle PQR$, expressing your answer as a fraction.

b) the ratio of $\sin \angle PQR : \sin \angle QPR$, expressing your answer as a ratio of two integers.

a)

$$\cos PQR = \frac{5^2 + 6^2 - 7^2}{2(5)(6)} = \frac{1}{5}$$

b)

$$\frac{\sin PQR}{7} = \frac{\sin QPR}{6}$$

$$\frac{\sin PQR}{\sin QPR} = \frac{7}{6}$$

$$\sin PQR : \sin QPR$$

$$7 : 6$$