

EQUITY

LEARNING PLACE

Elementary Math Topical (**Algebra**) Solutions

Question 1:

Simplify $3x - 7(2x + 1)$

$$3x - 7(2x + 1)$$

$$= 3x - 14x - 7$$

$$= -11x - 7$$

Question 2:

Factorise completely $2a^2 - 18$

$$2a^2 - 18$$

$$= 2(a^2 - 9)$$

$$= 2(a + 3)(a - 3)$$

Question 3:

It is given that $W = \frac{1}{2}m(v^2 - u^2)$

a) Calculate W when $m = 36.8$, $v = 2.05$ and $u = 0.39$

b) Express u in terms of W , m and v .

$$a) W = \frac{1}{2}(36.8)[(2.05)^2 - (0.39)^2]$$

$$W = 74.52736$$

$$b) W = \frac{1}{2}m(v^2 - u^2)$$

$$2W = mv^2 - mu^2$$

$$mu^2 = mv^2 - 2W$$

$$u^2 = \frac{mv^2 - 2W}{m}$$

$$u = \pm \sqrt{\frac{mv^2 - 2W}{m}}$$

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Question 4:

Write as a single fraction in its simplest form

$$\frac{2}{(x-3)^2} + \frac{3}{3-x}$$

$$\frac{2}{(x-3)^2} + \frac{3}{3-x}$$

$$= \frac{2}{(x-3)^2} + \frac{3(3-x)}{(3-x)^2}$$

$$= \frac{2}{(x-3)^2} + \frac{9-3x}{(3-x)^2}$$

$$= \frac{11-3x}{(x-3)^2}$$

Note: $(x-3)^2 = (3-x)^2$

Question 5:

Solve the following simultaneous equations

$$\frac{x}{3} - \frac{y}{4} = 2$$

$$\frac{x}{2} + \frac{y}{5} = 7.6$$

$$\frac{x}{3} - \frac{y}{4} = 2$$

$$4x - 3y = 24 \text{ — eqn 1}$$

$$\frac{x}{2} + \frac{y}{5} = 7.6$$

$$5x + 2y = 76 \text{ — eqn 2}$$

$$\text{eqn 1} \times 2: 8x - 6y = 48$$

$$\text{eqn 2} \times 3: 15x + 6y = 228$$

$$23x = 276$$

$$x = 12, y = 8$$

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Elementary Math Topical (**Algebra**) Solutions

Question 6:

a) Factorise completely $16 - 4x + xy - 4y$

b) Simplify

$$\frac{4}{x-5} - \frac{3x+9}{x^2-4x-5}$$

a) $16 - 4x + xy - 4y$

$$= 4(4 - x) + y(x - 4)$$

$$= 4(4 - x) - y(4 - x)$$

$$= (4 - y)(4 - x)$$

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Elementary Math Topical (Algebra) Solutions

Question 7:

Tap A, when turned on alone, can fill a tank completely in y minutes.

Tap B, when turned on alone, takes 12 minutes more to fill the same tank completely.

- What fraction of the tank is filled after one minute if only Tap A is turned on?
- What fraction of the tank is filled after one minute if only Tap B is turned on?
- Both taps are turned on at the same time. The tank is completely filled after 40 minutes. Form an equation in y and show that it reduces to $y^2 - 68y - 480 = 0$.
- Solve the equation $y^2 - 68y - 480 = 0$, giving your answers to 2 decimal places
- Given that Tap A has a water flow rate of 20ml per second, find the capacity of the tank in litres, correct to nearest litre.

a) $\frac{1}{y}$ tank

b) $\frac{1}{y+12}$ tank

c)

$$\frac{1}{y} + \frac{1}{y+12} = \frac{1}{40}$$

$$\frac{y+12+y}{y(y+12)} = \frac{1}{40}$$

$$40(2y+12) = y(y+12)$$

$$80y+480 = y^2+12y$$

$$y^2-68y-480 = 0$$

$$d) y = \frac{68 \pm \sqrt{(-68)^2 - 4(1)(-480)}}{2}$$

$$y = 74.4475 \text{ or } y = -6.4475$$

$$y \approx 74.45 \text{ or } y \approx -6.45$$

e) Capacity = $20 \times 60 \times 74.4475 = 89337 \text{ ml} \approx 89 \text{ l}$

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Elementary Math Topical (Algebra) Solutions

Question 8:

It is given that $a = \frac{3b-4c}{b+2c}$.

a) Find a when $b = 2$ and $c = -3$.

b) Express b in terms of a and c .

$$a) a = \frac{3b-4c}{b+2c}$$

$$a = \frac{3(2) - 4(-3)}{(2) + 2(-3)}$$

$$a = -\frac{9}{2}$$

$$b) a = \frac{3b-4c}{b+2c}$$

$$ab + 2ac = 3b - 4c$$

$$ab - 3b = -2ac - 4c$$

$$b(a - 3) = -2ac - 4c$$

$$b = \frac{-2ac - 4c}{a - 3}$$

Question 9:

Factorise completely

$$a) 2ax - 6x + ab - 3b.$$

$$b) 4x^2 + 7x - 2.$$

$$a) 2ax - 6x + ab - 3b$$

$$= 2x(a - 3) + b(a - 3)$$

$$= (a - 3)(2x + b)$$

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Question 10:

Simplify

a) $(2a+1)^2 - (a-1)^2$,

b) $\frac{3x}{2} - \frac{5(1-2x)}{3}$.

a) $(2a+1)^2 - (a-1)^2$
 $= [(2a+1+a-1)][(2a+1)-(a-1)]$
 $= (3a)(2a+1-a+1)$
 $= 3a(a+2)$

b) $\frac{3x}{2} - \frac{5(1-2x)}{3}$
 $= \frac{9x}{6} - \frac{10(1-2x)}{6}$
 $= \frac{9x - 10(1-2x)}{6}$
 $= \frac{9x - 10 + 20x}{6}$
 $= \frac{29x - 10}{6}$

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Elementary Math Topical (Algebra) Solutions

Question 11:

Write as a single fraction in its simplest form $\frac{2}{(x-3)^2} - \frac{1}{3-x}$

$$\frac{2}{(x-3)^2} - \frac{1}{3-x}$$

$$= \frac{2}{(x-3)^2} - \frac{3-x}{(3-x)^2}$$

$$= \frac{2 - (3-x)}{(x-3)^2}$$

$$= \frac{2 - 3 + x}{(x-3)^2}$$

$$= \frac{x-1}{(x-3)^2}$$

Note: $(x-3)^2 = (3-x)^2$

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Question 12:

Brandon and Ken took part in a 42 km marathon race. Ken ran at a constant speed of x km/h while Brandon ran at a constant speed which was 2 km/h less than Ken's speed.

Write down, in terms of x ,

- the time taken, in hours, that Ken took to complete the race,
- the time taken, in hours, that Brandon took to complete the race.
- Brandon took 20 more minutes longer to complete the race. Form an equation in x and show that it reduces to $x^2 - 2x - 252 = 0$.
- Solve the equation $x^2 - 2x - 252 = 0$, giving your solutions correct to three decimal places.
- Find the time taken for Brandon to complete the race. Give your answer in hours, minutes and seconds.

a) $\frac{42}{x}$ h

b) $\frac{42}{x-2}$ h

c)

$$\frac{42}{x-2} - \frac{42}{x} = \frac{20}{60}$$

$$\frac{42x - 42(x-2)}{x(x-2)} = \frac{1}{3}$$

$$\frac{84}{x^2 - 2x} = \frac{1}{3}$$

$$252 = x^2 - 2x$$

$$x^2 - 2x - 252 = 0$$

d) $x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-252)}}{2}$

$$x = 16.90597 \text{ or } x = -14.90597$$

$$x \approx 16.91 \text{ or } x \approx -14.91$$

e) $\frac{42}{16.90597-2} = 2.81766 \text{ h} = 2 \text{ h } 49 \text{ minutes } 4 \text{ seconds}$

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Elementary Math Topical (**Algebra**) Solutions

Question 13:

Solve the equation $\frac{2}{x+3} = 5x - 1$.

$$\frac{2}{x+3} = 5x - 1$$

$$2 = (5x - 1)(x + 3)$$

$$2 = 5x^2 + 14x - 3$$

$$5x^2 + 14x - 5 = 0$$

$$x = \frac{-14 \pm \sqrt{(14)^2 - 4(5)(-5)}}{2(5)}$$

$$x = 0.320 \text{ or } x = -3.12$$

Question 14:

A polar explorer is planning an expedition. He investigates three possible routes.

If he travels on route A, which is 700 km long, he expects to cover x km per day. Route B, which is the same distance as route A, has more difficult ice conditions and he would only expect to cover $(x - 5)$ km per day. Route C, which is 100 km longer than route A, has easier conditions and he would expect to cover $(x + 5)$ km per day.

Write down an expression, in terms of x , for the number of days that he expects to take on

a) route B,

b) route C.

c) He estimates that route C will take 10 days less than route B. Form an equation in x , and show that it reduces to $x^2 + 10x - 775 = 0$.

d) Solve the equation $x^2 + 10x - 775 = 0$, giving both answers correct to 2 decimal place.

e) Explain why one of the answers in (c) has to be rejected.

f) Calculate the minimum number of days the explorer will be away for his expedition if he takes on route C.

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Elementary Math Topical (**Algebra**) Solutions

Question 14 solution

a) $\frac{700}{x-5}$ day

b) $\frac{800}{x+5}$ day

c) $\frac{700}{x-5} - \frac{800}{x+5} = 10$

$$\frac{700(x+5) - 800(x-5)}{(x+5)(x-5)} = 10$$

$$700x + 3500 - 800x + 4000 = 10(x^2 - 25)$$

$$-100x + 7500 = 10x^2 - 250$$

$$10x^2 + 100x - 7750 = 0$$

$$x^2 + 10x - 775 = 0$$

$$d) x = \frac{-10 \pm \sqrt{(10)^2 - 4(1)(-775)}}{2}$$

$$x = 23.28427 \text{ or } x = -33.28427$$

$$x \approx 23.28 \text{ or } x \approx -33.28$$

e) Distance must be a positive real number

$$f) \frac{800}{23.28427+5} = 23.28$$

minimum number of days 24.

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Question 15:

a) Express $\frac{x+1}{2} + \frac{3x-2}{5}$ as a single fraction in its simplest form.

b) Expand and simplify $(4x-1)^2 - 3(8x+1)$.

c) Factorise completely $4q^2 - 4qr + pq - pr$.

$$\begin{aligned} \text{a) } & \frac{x+1}{2} + \frac{3x-2}{5} \\ &= \frac{5(x+1) + 2(3x-2)}{10} \\ &= \frac{5x+5+6x-4}{10} \\ &= \frac{11x+1}{10} \end{aligned}$$

$$\begin{aligned} \text{b) } & (4x-1)^2 - 3(8x+1) \\ &= 16x^2 - 8x + 1 - 24x - 3 \\ &= 16x^2 - 32x - 2 \end{aligned}$$

$$\begin{aligned} \text{c) } & 4q^2 - 4qr + pq - pr \\ &= 4q(q-r) + p(q-r) \\ &= (q-r)(4q+p) \end{aligned}$$

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Elementary Math Topical (**Algebra**) Solutions

Question 16:

p is a positive integer.

a) Explain why $(2p + 1)$ is an odd number.

b) Simplify the expression for the square of this odd number.

c) Hence explain why the square of an odd number is always an odd number.

a) $2p$ is an even number. The sum of any even number and 1 will be an odd numbers.

b) $(2p + 1)^2$

$$= 4p^2 + 4p + 1$$

c) $4p^2$ and $4p$ are even number. The sum of 2 even number and 1 will be an odd number.

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Elementary Math Topical (**Algebra**) Solutions

Question 17:

A swimming pool with a capacity of 1950 m^3 is fully filled with water.

a) Due to maintenance work, water is emptied from the pool at the rate of $x \text{ m}^3/\text{min}$. Express in terms of x , the time taken, in minutes, to empty all the water in the pool.

b) After maintenance, the empty pool is refilled with water at the rate of $(x - 2) \text{ m}^3/\text{min}$. Express in terms of x , the time taken, in minutes, to refill the swimming pool fully.

c) Given that it takes 15 minutes longer to refill the pool than to empty it, write down an equation to represent this information and show that it simplifies to

$$x^2 - 2x - 260 = 0.$$

d) Solve the equation $x^2 - 2x - 260 = 0$, giving your solutions correct to 3 decimal places.

e) Hence, find the time taken to empty the swimming pool, giving your answer in hours, minutes and seconds.

a) $\frac{1950}{x} \text{ min}$

b) $\frac{1950}{x-2} \text{ min}$

c) $\frac{1950}{x-2} - \frac{1950}{x} = 15$

$$\frac{1950x - 1950(x-2)}{x(x-2)} = 15$$

$$1950x - 1950x + 3900 = 15x^2 - 30x$$

$$15x^2 - 30x - 3900 = 0$$

$$x^2 - 2x - 260 = 0$$

d) $x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-260)}}{2}$

$$x = 17.15549 \text{ or } x = -15.15549$$

$$x \approx 17.155 \text{ or } x \approx -15.155$$

e) $\frac{1950}{17.155} = 113.666 \text{ mins} = 1 \text{ h } 53 \text{ mins } 40 \text{ s}$

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Elementary Math Topical (**Algebra**) Solutions

Question 18:

Simplify $(x - y)^2 - (x + y)^2$.

$$\begin{aligned}(x - y)^2 - (x + y)^2 \\ &= x^2 - 2xy + y^2 - (x^2 + 2xy + y^2) \\ &= x^2 - 2xy + y^2 - x^2 - 2xy - y^2 \\ &= -4xy\end{aligned}$$

(Note: Use of special identities is an alternate method)

Question 19:

It is given that $\frac{p}{x - p} = xy$. Express p in terms of x and y .

$$\begin{aligned}p &= xy(x - p) \\ p &= x^2y - pxy \\ p + pxy &= x^2y \\ p(1 + xy) &= x^2y \\ p &= \frac{x^2y}{1 + xy}\end{aligned}$$

Question 20:

Factorise completely $3x - 21y^2 + 9y - 7xy$.

$$\begin{aligned}3x + 9y - 21y^2 - 7xy \\ &= 3(x + 3y) - 7y(3y + x) \\ &= (x + 3y)(3 - 7y)\end{aligned}$$

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Elementary Math Topical (**Algebra**) Solutions

Question 21:

Express $\frac{7}{2x-1} - \frac{5}{2x^2+3x-2}$ as a single fraction, in its simplest form

$$= \frac{7}{2x-1} - \frac{5}{(2x-1)(x+2)}$$

$$= \frac{7(x+2) - 5}{(2x-1)(x+2)}$$

$$= \frac{7x+9}{(2x-1)(x+2)}$$

Question 22:

Wilfyrd owns a fruit stall. He sells apples at 50 cents each and oranges at 60 cents each. On a certain day, he sold x number of apples and y number of oranges. On that day, he sold 5 more apples than oranges. The total amount of money he received from selling the apples and oranges is \$32.20.

- a) From the information above, form two simultaneous equations relating the values of x and y .
- b) Solve the simultaneous equations in (i) to find the values of x and y .

a) $x - y = 5$

$$0.5x + 0.6y = 32.2$$

b) $x - y = 5$

$$5x - 5y = 25 - \text{eqn(1)}$$

$$0.5x + 0.6y = 32.2$$

$$5x + 6y = 322 - \text{eqn(2)}$$

eqn(2) - eqn(1):

$$(5x + 6y) - (5x - 5y) = 322 - 25$$

$$11y = 297$$

$$y = 27$$

$$x = 29$$

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Question 23:

Express $x^2 - 6x - 10$ in the form $(x + h)^2 + k$

Hence, solve $x^2 - 6x - 16 = 0$.

$$= (x - 3)^2 - 3^2 - 10$$

$$= (x - 3)^2 - 19$$

$$x^2 - 6x - 16 = 0$$

$$x^2 - 6x - 10 - 6 = 0$$

$$(x - 3)^2 - 19 - 6 = 0$$

$$(x - 3)^2 = 25$$

$$x - 3 = \pm 5$$

$$x = 8 \text{ or } x = -2$$

Question 24:

Simplify

a) $4(n - 2) - 3(n - 2)$

b) $g^2 - (g - 2h)(g + 3h)$

a) $4n - 8 - 3n + 6$

$$= n - 2$$

b) $g^2 - (g^2 + gh - 6h^2)$

$$= g^2 - g^2 - gh + 6h^2$$

$$= -gh + 6h^2$$

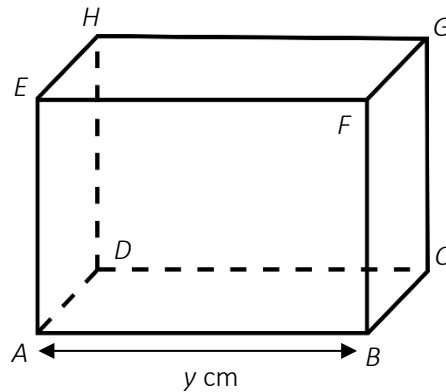
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Elementary Math Topical (**Algebra**) Solutions

Question 25:

The diagram below shows a cuboid of length y cm. The width of the cuboid is one third of its length. The height of the cuboid is 2 cm less than its length.



a) Write down an expression, in terms of y , for the width of the cuboid

b) Show that $AC = \frac{\sqrt{10}}{3}y$.

c) The length of the diagonal, AG , of the cuboid is $\sqrt{14}$ cm. Form an equation in y and show that it reduces to $19y^2 - 36y - 90 = 0$.

d) Solve the equation, giving your answers correct to 3 decimal places.

a) $\frac{1}{3}y$ cm

b) By Pythagoras' Theorem,

$$AC^2 = y^2 + \left(\frac{1}{3}y\right)^2$$

$$AC^2 = \frac{10}{9}y^2$$

$$AC = \frac{\sqrt{10}}{3}y$$

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c) $GC = y - 2$

$$AC^2 + GC^2 = AG^2$$

$$\frac{10}{9}y^2 + (y - 2)^2 = 14$$

$$10y^2 + 9y^2 - 36y + 36 = 126$$

$$19y^2 - 36y - 90 = 0$$

$$d) y = \frac{36 \pm \sqrt{(-36)^2 - 4(19)(-90)}}{2(19)}$$

$$y = 3.321 \text{ or } y = -1.426$$

Question 26:

a) Factorise completely $6am - 21an - 2m + 7n$.

b) Given that $9p^2 - 30pq + 25q^2 = 0$, find the ratio $p : q$.

c) Express as a single fraction in its simplest form $\frac{7x}{x^2 - 4y^2} + \frac{4}{2y - x}$.

a) $3a(2m - 7n) - 1(2m - 7n)$

$$= (2m - 7n)(3a - 1)$$

b) $(3p - 5q)(3p - 5q) = 0$

$$3p - 5q = 0$$

$$3p = 5q$$

$$\therefore p : q = 5 : 3$$

c) $\frac{7x}{(x+2y)(x-2y)} - \frac{4}{x-2y}$

$$= \frac{7x - 4(x + 2y)}{(x - 2y)(x + 2y)}$$

$$= \frac{3x - 8y}{(x - 2y)(x + 2y)}$$

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Elementary Math Topical (**Algebra**) Solutions

Question 27:

a) Solve $\frac{1}{x+2} - \frac{x-3}{x+4} = 5$

b) Express $x^2 - 6x - 2$ in the form $(x - p)^2 - q$.

a) $\frac{(x+4)-(x-3)(x+2)}{(x+2)(x+4)} = 5$

$$x + 4 - (x^2 - x - 6) = 5(x^2 + 6x + 8)$$

$$-x^2 + 2x + 10 = 5x^2 + 30x + 40$$

$$6x^2 + 28x + 30 = 0$$

$$3x^2 + 14x + 15 = 0$$

$$(3x + 5)(x + 3) = 0$$

$$x = -\frac{5}{3} \text{ or } x = -3$$

b) $(x - 3)^2 - 3^2 - 2$

$$= (x - 3)^2 - 11$$

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Elementary Math Topical (**Algebra**) Solutions

Question 28:

Cynthia bought x donuts for \$42.

a) Write down an expression, in terms of x , for the cost price, in cents, of one donut.

After keeping 8 donuts for her family, she sold the remaining donuts at 60 cents more than what she paid for it.

b) Write down an expression, in terms of x , for the total amount of money she received from the sale of the remaining donuts.

c) If Cynthia made a profit of \$6.60 from selling the remaining donuts, write down an equation in x to represent this information and show that it simplifies to $x^2 - 19x - 560 = 0$

d) Solve the equation $x^2 - 19x - 560 = 0$.

e) Find the selling price, in dollars, of each donut.

a) $\frac{4200}{x}$

b) $(x - 8) \left(\frac{4200}{x} + 60 \right)$

c) $(x - 8) \left(\frac{4200}{x} + 60 \right) = 4200 + 660$

$$4200 + 60x - \frac{33600}{x} - 480 = 4860$$

$$60x - \frac{33600}{x} = 1140$$

$$60x^2 - 1140x - 33600 = 0$$

$$x^2 - 19x - 560 = 0$$

d) $(x - 35)(x + 16) = 0$

$$x = 35 \text{ or } x = -16$$

e) Selling price = $\frac{4200}{35} + 60 = 180 \text{ cents} = \1.80

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Question 29:

a) Simplify $1 - (2x + 3)^2$.

b) Solve the equation $2x + 8 = \frac{2}{x}$, giving your answers correct to 2 decimal places.

$$a) 1 - (4x^2 + 12x + 9)$$

$$= 1 - 4x^2 - 12x - 9$$

$$= -4x^2 - 12x - 8$$

$$b) 2x + 8 = \frac{2}{x}$$

$$2x^2 + 8x - 2 = 0$$

$$x^2 + 4x - 1 = 0$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-1)}}{2}$$

$$x = 0.24 \text{ or } -4.24$$

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Question 30:

a) Simplify $\frac{t^2 - 25}{10 + 2t} \times \frac{2}{5 - t}$.

b) Express $\frac{4}{y - 2} + \frac{y}{y + 3}$ as a single fraction in its simplest form. Hence, solve the equation

$$\frac{4}{y - 2} + \frac{y}{y + 3} = 1.$$

c) Express $x^2 - 8x - 15$ in the form $(x + a)^2 + b$ where a and b are constants. Hence, solve the equation $x^2 - 8x - 15 = 0$.

$$\begin{aligned} \text{a) } & \frac{(t-5)(t+5)}{2(5+t)} \times \frac{2}{(-1)(t-5)} \\ & = -1 \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{4(y+3)+y(y-2)}{(y-2)(y+3)} \\ & = \frac{(4y + 12 + y^2 - 2y)}{(y - 2)(y + 3)} \\ & = \frac{y^2 + 2y + 12}{(y - 2)(y + 3)} \end{aligned}$$

$$\frac{y^2 + 2y + 12}{(y - 2)(y + 3)} = 1$$

$$y^2 + 2y + 12 = y^2 + y - 6$$

$$y = -18$$

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$$c) (x - 4)^2 - 4^2 - 15$$

$$= (x - 4)^2 - 31$$

$$(x - 4)^2 - 31 = 0$$

$$(x - 4)^2 - 31$$

$$x - 4 = \pm\sqrt{31}$$

$$x = 4 \pm \sqrt{31}$$

$$x = 9.57 \text{ or } x = -1.57$$