

EQUITY

LEARNING PLACE

Elementary Math Topical (**Prime Factors**)

Question 1:

a) Express 2025 as a product of its prime factors.

b) Using your answer in **(a)**, explain why 2025 is a perfect square.

c) a and b are both prime numbers. Find the values of a and b so that $2025 \times \frac{a}{b}$ is a perfect cube.

a) $2025 = 3^4 \times 5^2$

b) The index of each prime factors is multiple of 2 hence 2025 is a perfect square.

c) $a = 5, b = 3$

then $2025 \times \frac{5}{3} = 3^3 \times 5^3$, powers are of multiples of 3.

Question 2:

Tommy has 8100 one-centimetre cubes. He arranges all of the cubes into a cuboid.

The perimeter of the top of the cuboid is 70 cm. Each side of the cuboid has a length greater than 5 cm.

Find the height of the cuboid.

Volume = 8100 cm^3

Length, Breadth and height must be factors of 8100

$8100 = 2^2 \times 3^4 \times 5^2$

Perimeter = 70 cm implies that the sum of breadth and height is 35 cm

By listing and guessing,

$35 = 6 + 29 = 7 + 28 = 8 + 27 = 9 + 26 = 10 + 25 = 12 + 23 = 15 + 20$

Breadth = 15, length = 20

Height = $8100 \div 15 \div 20 = 27$

Therefore height is 27 cm

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Question 3:

- a) Express 324 as a product of its prime factors.
- b) Using your answer in **part (a)**, explain why 324 is a perfect square.
- c) m and n are both prime numbers. Find the values of m and n so that $324 \times \frac{m}{n}$ is a perfect cube.

a) $324 = 2^2 \times 3^4$

b) The index of each prime factor is multiple of 2 hence 324 is a perfect square.

c) $m = 2, n = 3$

Make the index multiple of 3.

Question 4:

Written as the product of its prime factors, $2205 = 3^2 \times 5 \times 7^2$.

- a) Express 40 as the product of its prime factors.
- Hence write down
- b) the LCM of 2205 and 40, giving your answer as the product of its prime factors.
- c) the smallest integer n such that $2205n$ is a perfect cube.

a) $40 = 2^3 \times 5$

b) $LCM = 2^3 \times 3^2 \times 5 \times 7^2$

c) $n = 3 \times 5^2 \times 7 = 525$

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Question 5:

Atiqah has 273 one-centimetre cubes. She uses all 273 cubes to make a cuboid. The sides of the cuboid are longer than 1 cm.

Find the dimensions of the cuboid.

Volume is 273 cm^3

$$273 = 3 \times 7 \times 13$$

The dimension is 3 cm by 7 cm by 13 cm.

Question 6:

a) It is given that $144 = 2^4 \times 3^2$. By expressing 216 as a product of prime factors in index notation, find the highest common factor of 144 and 216.

b) Find the smallest positive integer value of n such that $216n$ is a perfect square.

$$216 = 2^3 \times 3^3$$

$$HCF = 2^3 \times 3^2 = 72$$

$$b) n = 2 \times 3 = 6$$

Question 7:

Ernest has 3 pieces of ropes with lengths of 120 cm, 180 cm and 210 cm. He wishes to cut the 3 pieces of rope into small pieces of equal length without any remainder.

a) What is the greatest possible length of each small piece of rope cut?

b) How many small pieces of rope can he cut out altogether?

a) Find HCF for 120, 180, 210

$$120 = 2^3 \times 3 \times 5$$

$$180 = 2^2 \times 3^2 \times 5$$

$$210 = 2 \times 3 \times 5 \times 7$$

$$\text{Greatest possible length} = HCF = 2 \times 3 \times 5 = 30 \text{ cm}$$

$$b) \text{ Total number of pieces} = (120 \div 30) + (180 \div 30) + (210 \div 30) = 17$$

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Question 8:

Written as the product of its prime factors, $5292 = 2^2 \times 3^3 \times 7^2$.

- a) Write **378** as the product of its prime factors
- b) Given that **$378m$** is a perfect cube, write down the smallest possible integer value of **m** .
- c) Find two integers **a** and **b** other than 378 and 5292 such that their lowest common multiple is **5292** and their highest common factor is 378.

a) $378 = 2 \times 3^3 \times 7$

b) $378m = 2^3 \times 3^3 \times 7^3$

$\therefore m = 2^2 \times 7^2 = 196$

c)

$a = 2^2 \times 3^2 \times 7^2$

$b = 2^2 \times 3^2 \times 7^2$

$HCF = 2 \times 3^3 \times 7$

$LCM = 2^2 \times 3^3 \times 7^2$

By guessing and comparison,

$a = 2 \times 3^3 \times 7 = 378$

$b = 2^2 \times 3^3 \times 7^2 = 5292$

Or

$a = 2^2 \times 3^3 \times 7 = 756$

$b = 2 \times 3^3 \times 7^2 = 2646$

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Question 9:

a) Express 120 as a product of its prime factors.

It is given that $378 = 2 \times 3^3 \times 7$. Find

b) the highest common factor of 120 and 378.

c) the lowest common multiple of 120 and 378.

d) smallest integer k such that $378k$ is a multiple of 120.

e) the smallest integer p such that $378p$ is a perfect cube

a) $120 = 2^3 \times 3 \times 5$

b) $120 = 2^3 \times 3 \times 5$

$378 = 2 \times 3^3 \times 7$

$HCF = 2 \times 3 = 6$

c) $LCM = 2^3 \times 3^3 \times 5 \times 7 = 7560$

d) $k = 2^2 \times 5 = 20$

e) $p = 2^2 \times 7^2 = 196$

Question 10:

A cuboid of dimensions 12 cm \times 18 cm \times 24 cm is made of identical cubes. Find the least number of cubes required to form the cuboid.

$12 = 2^2 \times 3$

$18 = 2 \times 3^2$

$24 = 2^3 \times 3$

Length of a cube = $HCF = 2 \times 3 = 6$

Number of cubes = $(12 \times 18 \times 24) \div (6 \times 6 \times 6) = 24$

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Question 11:

a) Express 540 as the product of its prime factors.

b) Given that $540k$ is a perfect square, write down the smallest possible value of k .

$$a) 540 = 2^2 \times 3^3 \times 5$$

$$b) k = 3 \times 5 = 15$$

Question 12:

Schools A, B and C have their school bells ring at every 40 minutes, 45 minutes and at one hour interval respectively. If all the three schools ring at 9 am, at what time will the three schools ring together again?

$$40 = 2^3 \times 5$$

$$45 = 3^2 \times 5$$

$$60 = 2^2 \times 3 \times 5$$

$$LCM = 2^3 \times 3 \times 5 = 120 \text{ mins}$$

$$0900 + 120 \text{ mins} = 1100$$

Question 13:

Three warning lights flashed at intervals of 18, 21 and 28 seconds respectively. Given that they all start flashing together, find how many more times they flash together in the next hour.

$$18 = 2 \times 3^2$$

$$21 = 3 \times 7$$

$$28 = 2^2 \times 7$$

$$LCM = 2^2 \times 3^2 \times 7 = 252 \text{ seconds}$$

$$3600 \div 252 = 14.2851$$

\therefore in the next hour they will flash 14 times together.

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Question 14:

The numbers 180 and 784, written as the products of their prime factors, are $180 = 2^2 \times 3^2 \times 5^x$ and $784 = 2^4 \times 7^y$, where x and y are integers. Find

a) the values of x and y ,

b) the smallest positive integer value of q for which $180 \times 784 \times q$ is a perfect square.

$$a) 180 = 2^2 \times 3^2 \times 5$$

$$784 = 2^4 \times 7^2$$

$$\therefore x = 1, y = 2$$

$$b) 180 \times 784 \times q = 2^2 \times 3^2 \times 5 \times 2^4 \times 7^2 \times q = 2^6 \times 3^2 \times 5 \times 7^2 \times q$$

$$q = 5$$

Question 15:

Joe is tasked to plan for Sec One Orientation 2018. He wants to break all the students into small groups but he realises that if he puts them in pairs, there will be a student without partner. If he puts them in groups of 3 or 5 or 7, there will always be one group that is short of a student. Find the smallest possible total number of students.

Number of students is odd number since we cannot pair them.

Put in groups of 3 remainder will be 2, Put in groups of 5 remainder will be 4, Put in groups of 7 remainder will be 6

Listing

Group of 3 : 5, 8, 11, 14, 17, 20, 23, 26, 29

Group of 5 : 9, 14, 19, 24, 29

Common of group of 3 and 5 : 14, 29, 44, 59, 74, 89, 104, 119, 134, 149, 164, 179, 194, 209

Group of 5 : 9, 14, 19, 24, 29, 34

Group of 7 : 13, 20, 27, 34,

Common of group of 5 and 7: 34, 69, 104, 139, 174, 209

Total number of students is 209 students

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Question 16:

Alice has 165 one-centimetre cubes. She arranges all the cubes to form a cuboid with sides greater than 2 cm. Find the length of the longest side.

Volume is 165, this mean the length, breadth and height must be factors of 165

$$165 = 3 \times 5 \times 11$$

Length of longest side is 11 cm.

Question 17:

The prime factorisation of 144 is $2^4 \times 3^2$.

- Using the above information, explain why 144 is a perfect square.
- Find the highest common factor and lowest common multiple of 144 and 400.
- Find the largest possible integer m such that $\frac{144}{m}$ is a perfect cube greater than 1.

a) The indices of each prime number is multiple of 2, hence 144 is a prime number.

b) $400 = 2^4 \times 5^2$

$HCF = 2^4 = 16$

c) $m = 2 \times 3^2 = 18$

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Question 18:

When written as the product of their prime factors,

$$a \text{ is } 2^4 \times 5^2,$$

$$b \text{ is } 2^2 \times 5 \times 7,$$

$$c \text{ is } 2 \times 5^3 \times 7 \times 11.$$

Write down, as a product of its prime factors,

- the value of the square root of a ,
- the LCM of a , b , and c ,
- the greatest number that will divide a , b and c exactly.

$$\text{a) } \sqrt{a} = 2^2 \times 5$$

$$\text{b) } LCM = 2^4 \times 5^3 \times 7 \times 11$$

$$\text{c) } HCF = 2 \times 5$$

Question 19:

Express 378 as a product of its prime factors. Hence, find the value of n such that $378n$ is a perfect cube.

$$378 = 2 \times 3^3 \times 7$$

$$n = 2^2 \times 7^2 = 196$$

Question 20:

- Express 2160 as a product of its prime factors.
- Find the smallest number n such that $2160n$ is a perfect cube, where n is a positive integer.

$$\text{a) } 2160 = 2^4 \times 3^3 \times 5$$

$$\text{b) } n = 2^2 \times 5^2 = 100$$

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Question 21:

Ray has 120 one-centimetre cubes. He arranges all of the cubes into a cuboid. The perimeter of the top of the cuboid is 16 cm. Each side of the cuboid has a length greater than 2 cm. Find the height of the cuboid.

The volume is 120, the length, breadth and height must be factor of 120

$$120 = 2^3 \times 3 \times 5$$

Perimeter is 16 cm mean length + breadth is 8 cm

It could be 3+5 or 4+4

Since length and breadth must be factor of 120, length is 5 and breadth is 3

So height is 8 cm.

Question 22:

Three students begin a run at a starting line together at 17 30 hours. The students complete one round around the track in 100 seconds, 120 seconds and 160 seconds respectively. Assuming that the students maintain the same pace throughout the entire run, find the time when all three students next cross the starting line together.

$$100 = 2^2 \times 5^2$$

$$120 = 2^3 \times 3 \times 5$$

$$160 = 2^5 \times 5$$

$$LCM = 2^5 \times 3 \times 5^2 = 2400$$

$$1730 + 2400 \text{ seconds} = 1810$$

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Question 23:

a) Express **1936** as a product of its prime factors.

b) p and q are both **prime numbers**. Find the values of p and q so that $\frac{1936p}{q}$ is a perfect cube.

a) $1936 = 2^4 \times 11^2$

b) $p = 11, q = 2$

Question 24:

a) Express 60 and 825 as products of their prime factors and give your answers in index notation.

b) Find the smallest positive integer n for which $60n$ is a multiple of 825.

a) $60 = 2^2 \times 3 \times 5$

$825 = 3 \times 5^2 \times 11$

b) $n = 5 \times 11 = 55$

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Question 25:

As part of a Values-In-Action project, these items are packed into food hampers for distribution to the needy.

- 160 tins of milk powder
- 120 packets of instant cereal
- 100 packets of rice

Each hamper should have the same number of each food item. Find

a) the maximum number of food hampers,

b) the number of each food item in each hamper.

$$a) 160 = 2^6 \times 5$$

$$120 = 2^3 \times 3 \times 5$$

$$100 = 2^2 \times 5^2$$

$$HCF = 2^2 \times 5 = 20$$

$$b) \text{milk powder} = 8$$

$$\text{instant cereal} = 6$$

$$\text{rice} = 5$$

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Question 26:

- a) Express 63 as the product of its prime factors.
- b) What is the smallest positive integer value of n such that $63n$ is a multiple of 35?
- c) If $63m$ is a perfect cube, find the smallest possible integer value of m .
- d) A number p has exactly 6 factors. Two of the factors are 4 and 16. Find the value of p .

a) $63 = 3^2 \times 7$

b) $35 = 5 \times 7$

$n = 5$

c) $m = 3 \times 7^2 = 147$

d) If 4 and 16 are factors of p

Then 1, 2 are factors too. If 1 pair with 16 we have only 5 factors. So we pair 2 and 16

$2 \times 16 = 32$

So, $p = 32$