Additional Math Topical (Integration II)

Question 21:

a) Differentiate $\frac{-\cos x}{1+\sin x}$ with respect to *x*.

b) Hence, evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{5}}{1+\sin x}\,dx$, leaving your answers in surd form.

Question 22:

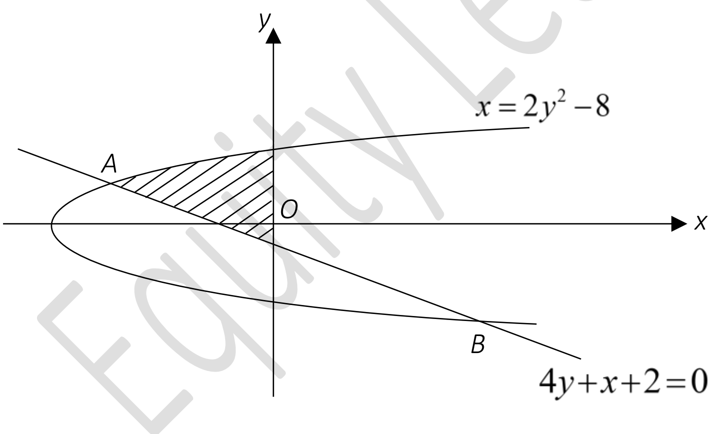
a) Factorise completely the cubic polynomial $2x^3 + 9x^2 + 13x + 6$.

b) Express $\frac{5x^2-3}{2x^3+9x^2+13x+6}$ as the sum of 3 partial fractions.

c) Hence or otherwise, find $\int \frac{5x^2-3}{2x^3+9x^2+13x+6}\,dx$

d) Show that $\int_0^1 \frac{5x^2-3}{2x^3+9x^2+13x+6}\,dx$ = *k*ln5 + *m*ln3 +*p*ln2, where *k*, *m*, *p* are real number

Question 23:



a) Find the coordinates of *A* and of *B*.

b) Find the area of the shaded region.

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Question 24:

A curve has the equation $y = 2(x-1)e^{2x}$.

a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = kxe^{2x} - 2e^{2x}$, where *k* is a constant.

b) Use your answer to part **(i)**, find the exact value of $\int_0^1 xe^{2x}\,\mathrm{d}x$.

Question 25:

It is given that $\int_2^4 a(3x-8)^4\,\mathrm{d}x = 1760$ where *a* is a constant.

a) Find the value of *a* and hence evaluate $\int_{\frac{4}{3}}^{3} a(3x-8)^4\,\mathrm{d}x$.

b) Express $\int_2^4 [a(3x-8)^4 + kx^2]\,\mathrm{d}x$ in terms of the constant *k*.

Question 26:

A particle moves in a straight line so that *t* seconds after passing through *O*, its velocity $v\,\mathrm{cm\,s^{-1}}$, is given by $v = t^2 - 6t + 5$. The particle comes to instantaneous rest, firstly at *A* and then at *B*.

a) Find the velocity of the particle when its acceleration is zero.

b) Find an expression, in terms of *t*, for the displacement of the particle, *s*, from *O* at time *t*.

c) Find the distance *AB*.

d) Find the total distance travelled in the first 8 seconds after passing through *O*

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Question 27:

a) Differentiate $y = 2x\sin x\cos x$ with respect to *x*.

b) Hence, find $\int_{\frac{\pi}{4}}^{0} \frac{x\cos 2x}{3}\,dx$.

Question 28:

a) Express $\frac{2x^3 - x^2 + 5x + 7}{x^2 - 4}$ in partial fractions.

b) Hence find $\int \frac{2x^3 - x^2 + 5x + 7}{x^2 - 4}\,dx$

Question 29:

The curve $y = 4 - e^x$ meets the *x*-axis at *P* and the *y*-axis at *Q*.

a) Find the coordinates of *P* and *Q*.

b) Sketch the curve of $y = 4 - e^x$.

c) Find the area of the region bounded by the curve and both axes.

d) Find the equation of the straight line which must be drawn on the graph of $y = 4 - e^x$ in order to obtain a solution to the equation $x = \ln(3 - x)$.

Question 30:

A function has the equation $y = (x-2)\sqrt{2x+1}$.

a) Show that $\frac{dy}{dx} = \frac{3x-1}{\sqrt{2x+1}}$

b) Hence, evaluate $\int_{2}^{5} \frac{6x-5}{\sqrt{2x+1}}\,dx$

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Question 31:

A curve is such that $\frac{d^2y}{dx^2} = 6x$.The line $9y = x + 44$ is a normal to the curve at the point $(1\,,5)$.

i) Show that the equation of the curve is $y = x^3 - 12x + 16$.

ii) Find the stationary points of the curve and their nature.

Question 32:

i) Given that $y = (x - 1)\sqrt{4x + 1}$, show that $\frac{dy}{dx} = \frac{ax+b}{\sqrt{4x+1}}$, where a and b are constants.

ii) Hence, evaluate

$$\int_1^4 \frac{12x - 5}{\sqrt{4x + 1}}\, dx$$

Question 33:

The variables x and y are related by the equation $y = 2x\sqrt{3x^2 + 5}$.

a) Express $\frac{\mathrm{dy}}{\mathrm{dx}}$ in the form $\frac{ax^2+b}{\sqrt{3x^2+5}}$, where a and b are integers.

b) evaluate

$$\int_0^2 \frac{18x^2 + 15}{\sqrt{3x^2 + 5}}\, \mathrm{d}x$$

Question 34:

Given that $\mathrm{f}(x) = \mathrm{e}^{-2x}\sin 2x + C$, $\mathrm{f}(0) = 2$ and

$\int \mathrm{e}^{-2x}\,\mathrm{g}(x)\mathrm{d}x = \mathrm{e}^{-2x}\sin 2x + C$.

a) Find the value of C.

b) Find an expression for $\mathrm{g}(x)$.

c) Hence, find $\int \mathrm{g}(x)\mathrm{d}x$.

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Question 35:

Find y as a function of x, given that $\frac{dy}{dx} = \frac{1}{3-2x}$

and $y = 2$ when $x = 1$.

Question 36:

Given that $f(x) = (x-1)e^{-x}$.

a) Find $f'(x)$.

b) Hence find $\int xe^{-x}\,dx$.

Question 37:

a) Express $\dfrac{7x^2-12x-3}{(x^2+3)(2x-3)}$ in partial fractions.

b) Differentiate $\ln(x^2+3)$ with respect to x.

c) Hence evaluate $\int_2^5 \dfrac{7x^2-12x-3}{(x^2+3)(2x-3)}\,dx$, giving your answer in the form $p\ln q$, where p and q are rational numbers.

Question 38:

A particle travels in a straight line such that, t seconds after passing a fixed point O, its velocity, v m/s, is given by $v = \dfrac{1}{\sqrt{1+t}} - \dfrac{1}{2}$.

a) Find the initial acceleration of the particle.

b) Find the value of t when the particle is at instantaneous rest.

c) Show that the distance travelled by the particle in the first 8 seconds is 1 m.

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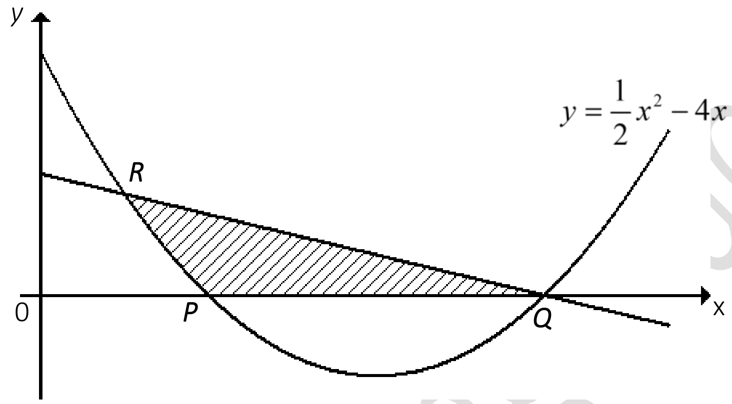
Question 39:

The diagram shows part of the curve $y=\frac{1}{2}x^2-4x+6$ which cuts intersects the x-axis at *P* (2, 0) and at *Q* (6, 0). The normal to the curve at *Q* meets the curve again at *R*.



a) Find the equation of the normal to the curve at *Q*.

b) Show that the coordinates of *R* is $\left(1, \frac{5}{2}\right)$.

c) Find the area of the shaded region

Question 40:

The equation of a curve is given by $y=\frac{3}{x}-\frac{8}{x^2}$.

a) Show that the tangent to the curve at the point $P\left(4, \frac{1}{4}\right)$ passes through the origin O.

b) The normal at the point $P\left(4, \frac{1}{4}\right)$ intersects the *x*-axis at *A*. Calculate the **exact** area of the triangle *OAP*.