1(i) $$\frac{15x^2+23x-7}{(2x-3)(x+2)^2} = \frac{A}{2x-3} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$15x^2+23x-7 = A(x+2)^2 + B(2x-3)(x+2) + C(2x-3)$$

when $x=-2$, when $x=\frac{3}{2}$, when $x=0$,

$7=-7C$ $61.25=12.25A$ $-7=4A-6B-3C$

$C=-1$ $A=5$ $6B=20+3+7$

$B=5$

$$\therefore \frac{15x^2+23x-7}{(2x-3)(x+2)^2} = \frac{5}{2x-3} + \frac{5}{x+2} - \frac{1}{(x+2)^2}$$

1(ii) $$\int \frac{15x^2+23x-7}{(2x-3)(x+2)^2}\,dx$$

$$= \int \frac{5}{2x-3} + \frac{5}{x+2} - \frac{1}{(x+2)^2}\,dx$$

$$= \frac{5\ln(2x-3)}{2} + \frac{5\ln(x+2)}{1} - \frac{(x+2)^{-1}}{-1} + C$$

$$= \frac{5}{2}\ln(2x-3) + 5\ln(x+2) + \frac{1}{x+2} + C$$

2a) $\frac{d}{dx}\left(\frac{x}{\sqrt{2-x}}\right)$

$$= \frac{(1)(2-x)^{\frac{1}{2}} - x\left(\frac{1}{2}\right)(2-x)^{-\frac{1}{2}}(-1)}{2-x}$$

$$= \frac{\frac{1}{2}(2-x)^{-\frac{1}{2}}\left[2(2-x)+x\right]}{2-x}$$

$$= \frac{4-x}{2(2-x)^{3/2}}$$

$$= \frac{4-x}{2\sqrt{(2-x)^3}}$$

2b) Eqn line AB :

$y = \frac{4-x}{\sqrt{(2-x)^3}}$ — ②

$0 = 4(-1)+c$

$c = 4$

$y = -x+4$ — ①

Sub ① into ②,

$$-x+4 = \frac{4-x}{\sqrt{(2-x)^3}}$$

$$(-x+4)\sqrt{(2-x)^3} = 4-x$$

$$(-x+4)\sqrt{(2-x)^3} + x - 4 = 0$$

2b) $(-x+4)\left[\sqrt{(2-x)^3}-1\right]=0$

$-x+4=0$ or $\sqrt{(2-x)^3}-1=0$

$x=4$ or $(2-x)^3=1$

$y=0$

$2-x=1$

$x=1$

$y=3$

$\therefore A(1,3)$

(ii) Area $= \frac{1}{2}(4+3)(1) - \int_0^1 \frac{4-x}{\sqrt{(2-x)^3}}\,dx$

$= \frac{7}{2} - 2\int_0^1 \frac{4-x}{2\sqrt{(2-x)^3}}\,dx$

$= \frac{7}{2} - 2\left[\frac{x}{\sqrt{2-x}}\right]_0^1$

$= \frac{7}{2} - 2\left[1-0\right]$

$= \frac{3}{2}$ units^2

3(i) $\frac{d}{dx}\left(\frac{1+3x}{\sqrt{3-2x}}\right)$

$$= \frac{3(3-2x)^{\frac{1}{2}} - (1+3x)(\frac{1}{2})(3-2x)^{-\frac{1}{2}}(-2)}{(3-2x)}$$

$$= \frac{(3-2x)^{-\frac{1}{2}}[3(3-2x)+(1+3x)]}{(3-2x)}$$

$$= \frac{10-3x}{\sqrt{(3-2x)^3}}$$

3ii) $\int_{-3}^{1} \frac{10-3x}{\sqrt{(3-2x)^3}}\,dx = \left[\frac{1+3x}{\sqrt{3-2x}}\right]_{-3}^{1}$

$$\int_{-3}^{1} \frac{10}{\sqrt{(3-2x)^3}}\,dx - \int_{-3}^{1} \frac{3x}{\sqrt{(3-2x)^3}}\,dx = 4-(-\frac{8}{3})$$

$$\left[\frac{10(3-2x)^{-\frac{1}{2}}}{(-2)(-\frac{1}{2})}\right]_{-3}^{1} - \frac{3}{5}\int_{-3}^{1} \frac{5x}{\sqrt{(3-2x)^3}}\,dx = \frac{20}{3}$$

$$10 - \frac{10}{3} - \frac{3}{5}\int_{-3}^{1} \frac{5x}{\sqrt{3-2x)^3}}\,dx = \frac{20}{3}$$

$$10 - \frac{10}{3} - \frac{20}{3} = \frac{3}{5}\int_{-3}^{1} \frac{5x}{\sqrt{(3-2x)^3}}\,dx$$

$$\int_{-3}^{1} \frac{5x}{\sqrt{(3-2x)^3}}\,dx = 0$$

4a) $\frac{2x^2+1}{(x+2)(x+3)}$

$$x^2+5x+6 \overset{2}{\overline{\left) 2x^2+1 \right.}}$$
$$\underline{2x^2+10x+12}$$
$$-10x-11$$

$$= 2 + \frac{-10x-11}{(x+2)(x+3)}$$

$$\frac{-10x-11}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$$

$$-10x-11 = A(x+3) + B(x+2)$$

Let $x=-3$, Let $x=-2$,

$19 = -B$ $9 = A$

$B = -19$

$$\therefore \frac{2x^2+1}{(x+2)(x+3)} = 2 + \frac{9}{x+2} - \frac{19}{x+3}$$

4b) $\int \frac{2x^2+1}{(x+2)(x+3)} dx$

$$= \int 2 + \frac{9}{x+2} - \frac{19}{x+3} dx$$

$$= 2x + 9\ln(x+2) - 19\ln(x+3) + C$$

5a) $\int \tan^2(2x)\, dx$

$= \int \sec^2(2x) - 1 \; dx$

$= \frac{\tan(2x)}{2} - x + C$

5b) $\int_0^{\frac{\pi}{4}} \tan^2(2x)\, dx$

$= \left[\frac{\tan(2x)}{2} - x \right]_0^{\frac{\pi}{4}}$

It cannot be evaluated because $\tan(2x)$ is undefined when $x = \frac{\pi}{4}$.

6)
$$\int_2^6 \frac{2}{x^3} + \frac{1}{\sqrt{2x-1}}\,dx$$

$$= \left[\frac{2x^{-2}}{-2} + \frac{(2x-1)^{\frac{1}{2}}}{(\frac{1}{2})(2)} \right]_2^6$$

$$= \left[-\frac{1}{x^2} + \sqrt{2x-1} \right]_2^6$$

$$= 1.80679$$

$$\approx 1.81$$

7(i) $y = \frac{3x-4}{\sqrt{2x-1}}$

$$\frac{dy}{dx} = \frac{3(2x-1)^{\frac{1}{2}} - (3x-4)(\frac{1}{2})(2x-1)^{-\frac{1}{2}}(2)}{2x-1}$$

$$= \frac{(2x-1)^{-\frac{1}{2}}[3(2x-1)-(3x-4)]}{2x-1}$$

$$= \frac{3x+1}{\sqrt{(2x-1)^3}}$$

7(ii) $\int \frac{1}{\sqrt{(2x-1)^3}}\,dx$

$$= \frac{(2x-1)^{-\frac{1}{2}}}{(-\frac{1}{2})(2)} + C$$

$$= -\frac{1}{\sqrt{2x-1}} + C$$

7iii)

$$\int \frac{3x+1}{\sqrt{(2x-1)^3}}\,dx = \frac{3x-4}{\sqrt{2x-1}} + c$$

$$\int \frac{3x}{\sqrt{(2x-1)^3}}\,dx + \int \frac{1}{\sqrt{(2x-1)^3}}\,dx = \frac{3x-4}{\sqrt{2x-1}} + c$$

$$3\int \frac{x}{\sqrt{(2x-1)^3}}\,dx = \frac{3x-4}{\sqrt{2x-1}} - \int \frac{1}{\sqrt{(2x-1)^3}} + c$$

$$3\int \frac{x}{\sqrt{(2x-1)^3}}\,dx = \frac{3x-4}{\sqrt{2x-1}} + \frac{1}{\sqrt{2x-1}} + c$$

$$\int \frac{x}{\sqrt{(2x-1)^3}}\,dx = \frac{x-1}{\sqrt{2x-1}} + c$$

8i) $V = t^2 - 6t + 5$

when $t = 0$,

$V = 5$

8ii) when $v = 0$,

$t^2 - 6t + 5 = 0$

$(t-5)(t-1) = 0$

$t = 5$ or $t = 1$

8iii) $a = \frac{dV}{dt} = 2t - 6$

when $t = 2$,

$a = -2$

8iv) min velocity $\Rightarrow \frac{dV}{dt} = 0$

$2t - 6 = 0$

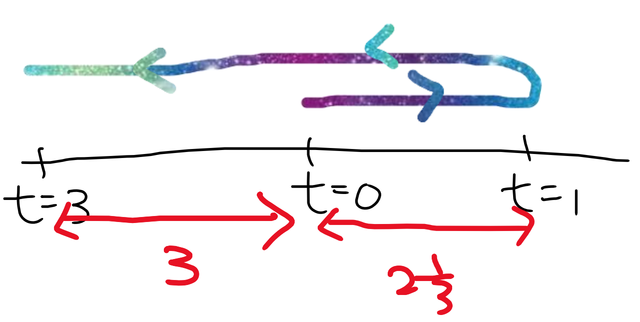
$t = 3$

$V = (3)^2 - 6(3) + 5$

$= -4$

$\frac{d^2V}{dt^2} = 2 > 0$

∴ min. velocity is -4 m/s



8v)

$S = \int v \, dt$

$= \int t^2 - 6t + 5 \, dt$

$= \frac{t^3}{3} - 3t^2 + 5t + C$

when $S = 0$, $t = 0$

$C = 0$

∴ $S = \frac{t^3}{3} - 3t^2 + 5t$

when $t = 1$,

$S = \frac{1}{3} - 3 + 5$

$= 2\frac{1}{3}$

when $t = 3$,

$S = 9 - 27 + 15$

$= -3$

total distance

$= 2\frac{1}{3} + 2\frac{1}{3} + 3$

$= 7\frac{2}{3}$ m

9i) . $y = e^x$ — ①

$y = 4 - e^x$ — ②

Sub ① into ②,

$e^x = 4 - e^x$

$e^x = 2$

$x = \ln 2$

$y = e^{\ln 2}$

$= 2$

$\therefore P(\ln 2, 2)$

From ②,

when $y = 0$,

$e^x = 4$

$x = \ln 4$

$\therefore B(\ln 4, 0)$

$\frac{dy}{dx} = e^x$

when $x = \ln 2$,

$\frac{dy}{dx} = 2$

$2 = 2\ln 2 + c$

$c = 2 - 2\ln 2$

$\therefore$ Eqn PQ :

$y = 2x + 2 - 2\ln 2$

9ii)

$$\text{Area OBPA} = \int_0^{\ln 2} e^x \, dx + \int_{\ln 2}^{\ln 4} 4 - e^x \, dx$$

$$= \left[e^x\right]_0^{\ln 2} + \left[4x - e^x\right]_{\ln 2}^{\ln 4}$$

$$= 1 + \left[4\ln 4 - 4 - 4\ln 2 + 2\right]$$

$$= 4\ln 2 - 1$$

$$\text{Area of water} = \int_0^{\ln 2} e^x dx - \frac{1}{2}(2 - 2\ln 2 + 2)(\ln 2)$$

$$= 1 - 2\ln 2 + (\ln 2)^2$$

$$\% \text{ of water} = \frac{1 - 2\ln 2 + (\ln 2)^2}{4\ln 2 - 1} \times 100$$

$$= 5.3119 \%$$

$$\approx 5.31\%$$

10a) $\frac{d}{dx}\left[\ln(x+1)^x\right]$

$= \frac{d}{dx}\left[x\ln(x+1)\right]$

$= \ln(x+1) + x\left(\frac{1}{x+1}\right) + c$

$= \ln(x+1) + \frac{x}{x+1} + c$

10bi) $\int_0^{\frac{\pi}{2}} f(x)\,dx = \left[\cos x + k\sin 3x\right]_0^{\frac{\pi}{2}}$

$5 = -k - 1$

$k = -6$

10bii) $\int f(x)\,dx = \cos x - 6\sin 3x + c$

$\frac{d}{dx}\int f(x)\,dx = \frac{d}{dx}\left(\cos x - 6\sin 3x + c\right)$

$f(x) = -\sin x - \frac{6\cos 3x}{3}$

$= -\sin x - 2\cos 3x$

10b(iii) $\frac{dy}{dx} = -\cos x + 6\sin 3x$

when $x = \frac{\pi}{2}$,

$\frac{dy}{dx} = -6$, $y = -1$

grad. of normal $= \frac{1}{6}$

$-1 = \frac{1}{6}\left(\frac{\pi}{2}\right) + c$

$c = -1 + \frac{\pi}{12}$

$\therefore$ eqn. of normal :

$y = \frac{1}{6}x + \frac{\pi}{12} - 1$

11a) $\frac{d}{dx}\left(2x\sqrt{1-x^2} \right)$

$$= 2(1-x^2)^{\frac{1}{2}} + 2x\left(\tfrac{1}{2}\right)(1-x^2)^{-\frac{1}{2}}(-2x)$$

$$= (1-x^2)^{-\frac{1}{2}}\left[2(1-x^2) - 2x^2 \right]$$

$$= \frac{2-4x^2}{\sqrt{1-x^2}}$$

11b) $\int \frac{2-4x^2}{\sqrt{1-x^2}}\,dx = 2x\sqrt{1-x^2} + c$

$$\frac{2}{5}\int \frac{5-10x^2}{\sqrt{1-x^2}}\,dx = 2x\sqrt{1-x^2} + C$$

$$\int \frac{5-10x^2}{\sqrt{1-x^2}}\,dx = 5x\sqrt{1-x^2} + c$$

12a)

$$\frac{3-2x}{(1-2x)^2} = \frac{A}{1-2x} + \frac{B}{(1-2x)^2}$$

$$3-2x = A(1-2x) + B$$

$$3-2x = A - 2Ax + B$$

$$\therefore A=1, \quad B=2$$

$$\frac{3-2x}{(1-2x)^2} = \frac{1}{1-2x} + \frac{2}{(1-2x)^2}$$

12b)

$$\int \frac{3-2x}{(1-2x)^2}\,dx$$

$$= \int \frac{1}{1-2x} + \frac{2}{(1-2x)^2}\,dx$$

$$= \frac{\ln(1-2x)}{-2} + \frac{2(1-2x)^{-1}}{(-1)(-2)} + C$$

$$= -\frac{1}{2}\ln(1-2x) + \frac{1}{1-2x} + C$$

13) $s = t^3 - 14t^2 + 60t + 8$

a) $v = \frac{ds}{dt} = 3t^2 - 28t + 60$

when $v = 0$,

$3t^2 - 28t + 60 = 0$

$(3t - 10)(t - 6) = 0$

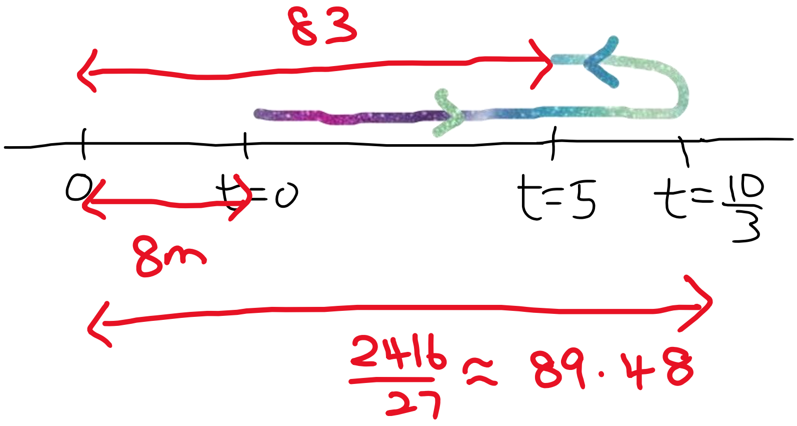
$t = \frac{10}{3}$ or $t = 6$

13b) when $t = \frac{10}{3}$,

$S = \frac{2416}{27}$

when $t = 5$,

$S = 83$



13c) when $t = 6$,

$S = 80$,

It will not because it will turn away from the starting position 80m from O and it will not turn again.

Total dist travelled

$\left(\frac{2416}{27} - 8\right) + \left(\frac{2416}{27} - 83\right) = 87.963$

≈ 88.0 m

14a) $y = 9-(x-3)^2$

$y = 9-(x^2-6x+9)$

$y = -x^2+6x$

$\frac{dy}{dx} = -2x+6$

when $x=5$,

$\frac{dy}{dx} = -4$, $y = 5$

$5 = -4(5)+c$

$c = 25$

Eqn PQ : $y = -4x+25$

when $y=0$,

$4x = 25$

$x = \frac{25}{4}$

$\therefore Q\left(\frac{25}{4},0\right)$

14b) when $y=0$

$-x^2+6x = 0$

$x(-x+6) = 0$

$x=0$ or $x=6$

Shaded area

$= \frac{1}{2}(5)\left(\frac{25}{4}-5\right) - \int_5^6 -x^2+6x\,dx$

$= \frac{45}{8} - \left[-\frac{x^3}{3}+3x^2\right]_5^6$

$= \frac{45}{8} - \left[-\frac{216}{3}+108+\frac{125}{3}-75\right]$

$= \frac{71}{24}$ units^2

15a) $\int_{-1}^{3} [f(x)+1]\,dx = 12$

$$\int_{-1}^{3} f(x)\,dx + \int_{-1}^{3} 1\,dx = 12$$

$$\int_{-1}^{3} f(x)\,dx + [x]_{-1}^{3} = 12$$

$$\int_{-1}^{3} f(x)\,dx + 4 = 12$$

$$\int_{1}^{3} f(x)\,dx = 8$$

15b) $\int_{2}^{3} [f(x)+1]\,dx + \int_{-1}^{2} [f(x)+1]\,dx$

$$= \int_{-1}^{3} [f(x)+1]\,dx$$

$$= 12$$

16a) $\frac{d}{dx}\tan^3 x$

$= 3(\tan x)^2(\sec^2 x)$

$= 3\tan^2 x \sec^2 x$

$= 3(\sec^2 x - 1)(\sec^2 x)$

$= 3\sec^4 x - 3\sec^2 x$

16b) $\int_0^{\frac{\pi}{3}} 3\sec^4 x - 3\sec^2 x \, dx = \left[\tan^3 x\right]_0^{\frac{\pi}{3}}$

$\int_0^{\frac{\pi}{3}} 3\sec^4 x \, dx - \int_0^{\frac{\pi}{3}} 3\sec^2 x \, dx = 3\sqrt{3}$

$3\int_0^{\frac{\pi}{3}} \sec^4 x \, dx - \left[3\tan x\right]_0^{\frac{\pi}{3}} = 3\sqrt{3}$

$3\int_0^{\frac{\pi}{3}} \sec^4 x \, dx = 3\sqrt{3} + 3\sqrt{3}$

$\int_0^{\frac{\pi}{3}} \sec^4 x \, dx = 2\sqrt{3}$

17a) $\frac{x+1}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$

$$x+1 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$$

when $x=-2$

$-1 = -3C$

$C = \frac{1}{3}$

when $x=1$,

$2 = 9A$

$A = \frac{2}{9}$

when $x = 0$

$1 = 4A - 2B - C$

$1 = \frac{8}{9} - 2B - \frac{1}{3}$

$B = -\frac{2}{9}$

$$\therefore \frac{x+1}{(x-1)(x+2)^2} = \frac{2}{9(x-1)} - \frac{2}{9(x+2)} + \frac{1}{3(x+2)^2}$$

17b) $\int_2^4 \frac{x+1}{(x-1)(x+2)^2}\,dx$

$$= \int_2^4 \frac{2}{9(x-1)} - \frac{2}{9(x+2)} + \frac{1}{3(x+2)^2}\,dx$$

$$= \left[\frac{2}{9}\ln(x-1) - \frac{2}{9}\ln(x+2) + \frac{1}{3}\frac{(x+2)^{-1}}{-1}\right]_2^4$$

$$= \left[\frac{2}{9}\ln 3 - \frac{2}{9}\ln 6 - \frac{1}{18}\right] - \left[0 - \frac{2}{9}\ln 4 - \frac{1}{12}\right]$$

$$= 0.181810$$

$$\approx 0.182$$

18) $x > \frac{1}{2}$

$2x > 1$

$2x - 1 > 0$

$\sqrt{4x-1} > 0$

$\therefore \quad (2x-1)\sqrt{4x-1} > 0$

$f'(x) > 0$

$f'(x)$ is an increasing function.

18b) $f''(x) = 2(4x-1)^{\frac{1}{2}} + (2x-1)(\frac{1}{2})(4x-1)^{-\frac{1}{2}}(4)$

$= (4x-1)^{-\frac{1}{2}}[2(4x-1) + 2(2x-1)]$

$$= \frac{12x-4}{\sqrt{4x-1}}$$

18c) $\int \frac{12x-4}{\sqrt{4x-1}}\,dx = (2x-1)(\sqrt{4x-1}) + c$

$\int \frac{12x}{\sqrt{4x-1}}\,dx - \int \frac{4}{\sqrt{4x-1}}\,dx = (2x-1)(\sqrt{4x-1}) + c$

$\int \frac{12}{\sqrt{4x-1}}\,dx = (2x-1)(\sqrt{4x-1}) + \int \frac{4}{\sqrt{4x-1}}\,dx + c$

$\int \frac{12}{\sqrt{4x-1}}\,dx = (2x-1)(\sqrt{4x-1}) + \frac{4(4x-1)^{\frac{1}{2}}}{(4)(\frac{1}{2})} + c$

$\int \frac{12}{\sqrt{4x-1}}\,dx = (2x-1)(\sqrt{4x-1}) + 2\sqrt{4x-1} + c$

18a)

$$g'(x) = \frac{12x}{\sqrt{4x-1}}$$

$$g(x) = \int \frac{12x}{\sqrt{4x-1}}\,dx$$

$$g(x) = (2x-1)\sqrt{4x-1} + 2\sqrt{4x-1} + c$$

when $x = \frac{5}{4}$, $g(x) = 10$

$$10 = \frac{3}{2}(2) + 2(2) + c$$

$$c = 3$$

$$\therefore g(x) = (2x+1)\sqrt{4x-1} + 3$$

19a) $\int_1^4 ax\,dx = 5$

$\left[\frac{ax^2}{2}\right]_1^4 = 5$

$8a - \frac{1}{2}a = 5$

$a = \frac{2}{3}$

$\int_2^5 \frac{2}{3}x\,dx$

$= \left[\frac{1}{3}x^2\right]_2^5$

$= \frac{25}{3} - \frac{4}{3}$

$= 7$

19b) $\int_1^4 (\frac{2}{3}x + b)\,dx$

$= \left[\frac{1}{3}x^2 + bx\right]_1^4$

$= \frac{16}{3} + 4b - \frac{1}{3} - b$

$= 5 + 3b$

20) $\frac{d^2y}{dx^2} = \frac{9}{(1+3x)^2}$

$$\frac{dy}{dx} = \int \frac{9}{(1+3x)^2} dx$$

$$= \frac{9(1+3x)^{-1}}{(-1)(3)} + c$$

$$= -\frac{3}{1+3x} + c$$

when $\frac{dy}{dx} = 0$, $x = 0$

$$0 = -3 + c$$

$$c = 3$$

$$\frac{dy}{dx} = -\frac{3}{1+3x} + 3$$

$$y = \int \frac{-3}{1+3x} + 3 \, dx$$

$$= \frac{-3\ln(1+3x)}{3} + 3x + c$$

when $x = 0$, $y = 5$,

$$5 = c$$

$$\therefore y = -\ln(1+3x) + 3x + 5$$