EQUITY
LEARNING PLACE

Additional Math Topical (Integration I)

Question 1:

i) Express the following in partial fractions

$$\frac{15x^2 + 23x - 7}{(2x - 3)(x + 2)^2}$$

ii) Hence, find

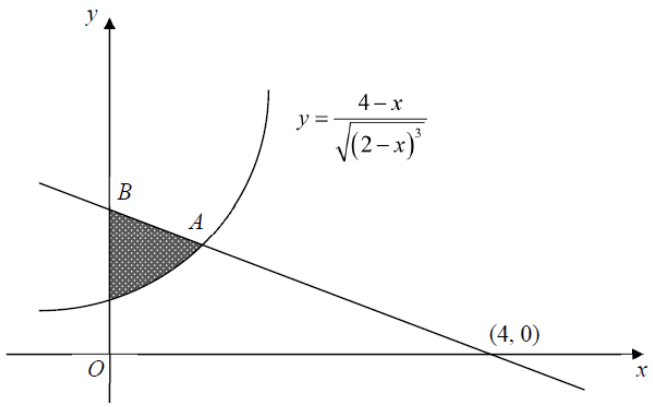
$$\int \frac{15x^2 + 23x - 7}{(2x - 3)(x + 2)^2}\, dx$$

Question 2:

a) Show that

$$\frac{d}{dx}\left(\frac{x}{\sqrt{2 - x}}\right) = \frac{4 - x}{2\sqrt{(2 - x)^3}}$$

b)



The diagram shows part of the curve $y = \frac{4-x}{\sqrt{(2-x)^3}}$. The line through the point $(4\,,0)$ with gradient -1 intersects the curve and the $y-$ axis at points A and B respectively.

i) Show that A has the coordinates $(1\,,3)$

ii) Using the results of parts (a) and (bi), find the area of the shaded region bounded by the curve, the line AB and the $y-$ axis.

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Question 3:

i) Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1+3x}{\sqrt{3-2x}}\right)=\frac{a+bx}{\sqrt{(3-2x)^3}}$, where *a* and *b* are integers.

ii) Hence evaluate $\int_{-3}^{1}\frac{5x}{\sqrt{(3-2x)^3}}\mathrm{d}x$.

Question 4:

a) Express $\frac{2x^2+1}{(x+2)(x+3)}$ in partial fractions.

b) Hence, find $\int\frac{2x^2+1}{(x+2)(x+3)}\,\mathrm{d}x$.

Question 5:

a) Find $\int\tan^2(2x)\,\mathrm{d}x$.

b) Explain whether $\int_0^{\frac{\pi}{4}}\tan^2(2x)\,\mathrm{d}x$ can be evaluated.

Question 6:

Evaluate $\int_2^6\frac{2}{x^3}+\frac{1}{\sqrt{2x-1}}\,\mathrm{d}x$.

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Question 7:

i) Given that $y = \frac{3x-4}{\sqrt{2x-1}}$, express $\frac{dy}{dx}$ in the form $\frac{ax+b}{\sqrt{(2x-1)^3}}$, where a and b are constants.

ii) Find $\int \frac{1}{\sqrt{(2x-1)^3}}\, dx$

iii) Using the result from part (i) and (ii), find

$$\int \frac{x}{\sqrt{(2x-1)^3}}\, dx$$

Question 8:

A particle moves in a straight line so that t seconds after leaving a fixed point O, its velocity, $v\ \text{ms}^{-1}$, is given by

$$v = t^2 - 6t + 5$$

Find

i) the initial velocity of the particle.

ii) the values of t when the particle is at rest.

iii) the acceleration of the particle when $t = 2$

iv) the minimum velocity of the particle

v) the total distance travelled in the first 3 seconds.

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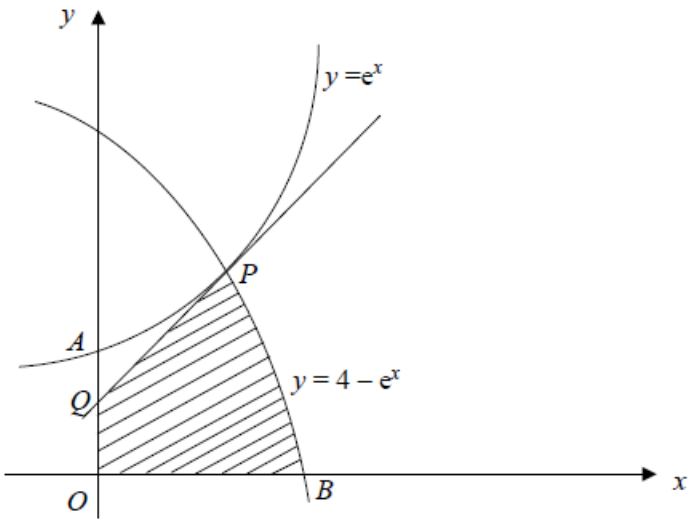
Question 9:

The region $OAPB$ represents a landscaped plot. The shaded region is to be turfed with carpet grass while the unshaded region is reserved for a water feature. The side AP is represented by the curve $y = e^x$ and the side PB is represented by the curve $y = 4 - e^x$. The line PQ is a tangent to the curve $y = e^x$ at P.



i) Find the equation of the line PQ.

ii) Find the percentage of the plot that is reserved for the water feature.

Question 10:

a) Find $\frac{d}{dx}[\ln(x+1)^x]$

b) It is given that $\int \mathrm{f}(x)dx = \cos x + k \sin 3x + c$, where c is a constant and that $\int_0^{\frac{\pi}{2}} \mathrm{f}(x)dx = 5$.

i) Show that $k = -6$

ii) Find $\mathrm{f}(x)$

iii) Find the equation of the normal to the curve $y = \mathrm{f}(x)$ at the point where $x = \frac{\pi}{2}$

Additional Math Topical (Integration I)

Question 11:

a) Show that $\frac{d}{dx}\left(2x\sqrt{1-x^2}\right) = \frac{2-4x^2}{\sqrt{1-x^2}}$.

b) Hence find $\int \frac{5-10x^2}{\sqrt{1-x^2}}dx$.

Question 12:

a) Express $\frac{3-2x}{(1-2x)^2}$ in partial fractions.

b) Hence find $\int \frac{3-2x}{(1-2x)^2}dx$.

Question 13:

A particle P moves in a straight line so that its displacement, s m, from a fixed point O is given by $s = t^3 - 14t^2 + 60t + 8$.

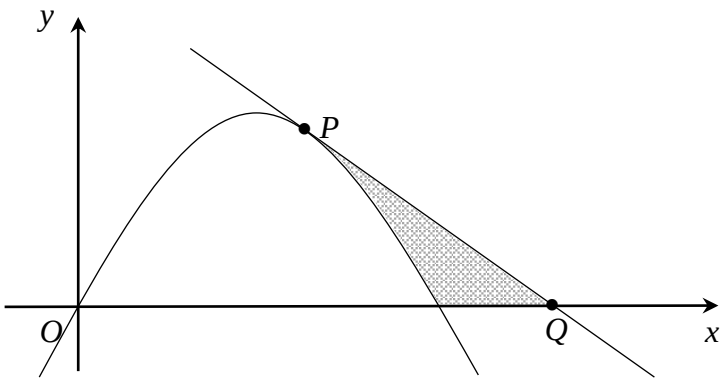
a) Find the values of t when P is at instantaneous rest.

b) Find the total distance travelled in the first 5 seconds.

c) Determine whether P will return to its starting position.

Additional Math Topical (Integration I)

Question 14:



The diagram shows part of the curve $y = 9 - (x - 3)^2$ and the point P lies on the curve where $x = 5$. Given that the tangent to the curve at P cuts the x–axis at Q.

a) Find the coordinates of Q.

b) Find the area of the shaded region bounded by the curve, the line PQ and the x–axis.

Question 15:

Given that $\int_{-1}^{3}[f(x)+1]dx = 12$, evaluate

a) $\int_{-1}^{3} 2f(x)dx$,

b) $\int_{2}^{3}[f(x)+1]dx + \int_{-1}^{2}[f(x)+1]dx$.

Question 16:

a) Differentiate $\tan^3 x$ with respect to *x*.

b) Hence, evaluate $\int_{0}^{\frac{\pi}{3}} \sec^4 x \, dx$, leaving your answer in exact form.

Additional Math Topical (Integration I)

Question 17:

a) Express $\frac{x+1}{(x-1)(x+2)^2}$ in partial fractions.

b) **Hence,** evaluate $\int_2^4 \frac{x+1}{(x-1)(x+2)^2}\,dx$.

Question 18:

It is given that $f(x)$ is defined for $x > \frac{1}{2}$ and is such that $f'(x) = (2x-1)\sqrt{4x-1}$.

a) Explain whether $f(x)$ is an increasing or a decreasing function.

b) Show that $f''(x)$ can be expressed in the form $\frac{ax+b}{\sqrt{4x-1}}$.

c) Integrate $\frac{12x}{\sqrt{4x-1}}$ with respect to x.

d) Given that the curve $y = g(x)$ passes through the point $\left(\frac{5}{4}, 10\right)$ and

$g'(x) = \frac{12x}{\sqrt{4x-1}}$, find an expression for $y = g(x)$.

Question 19:

It is given that $\int_1^4 ax\ dx = 5$, where a is a constant.

a) Find the value of $\int_2^5 ax\ dx$.

b) Express $\int_1^4 (ax + b)dx$ in terms of the constant b.

Additional Math Topical (Integration I)

Question 20:

A curve is such that $\frac{d^2y}{dx^2} = \frac{9}{(1+3x)^2}$. Given that the curve has a stationary point at (0, 5), find the equation of the curve.