Equity Learning Place

Additional Math Topical (Differentiation II)

Question 21:

Given $y = 4\cos^3 x - 3\cos x$,

a) Find the value of $\frac{dy}{dx}$ at the point when $x = \frac{\pi}{6}$.

b) Find the equation of the normal passing through the point $x = \frac{\pi}{6}$.

Question 22:

A curve has the equation $y = \frac{2x+5}{x-2}$, $x \neq 2$.

a) Find the gradient of the curve at the point where it crosses the x-axis.

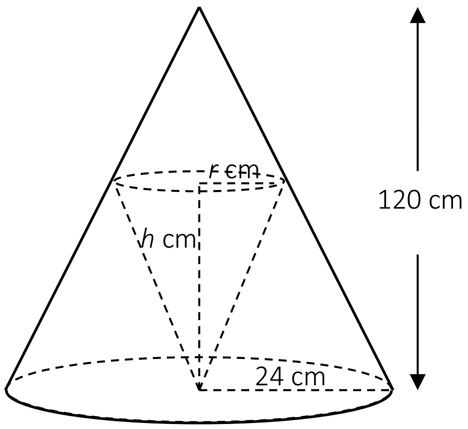
b) State whether this curve has a turning point. Justify your answer.

c) Given that a point (x, y) moves along the curve in such a way that the y coordinate is decreasing at a constant rate of 0.02 units per second.

Find the corresponding rate of the change of the x coordinate at the instant when $x = 3$.

Question 23:

The diagram shows a solid cone with base radius 24 cm and height 120 cm.



A cone of radius r cm and height h cm is to be removed as shown.

a) Express h in terms of r.

b) Hence, show that the volume, V cm^3, of the cone to be removed is given by

$$V = 40\pi r^2 - \frac{5\pi r^3}{3}.$$

c) Calculate the value of r for which V has a stationary value.

Hence, find the stationary value of V and determine whether it is a maximum or minimum value.

Question 24:

It is given that $y=(4x+1)\sqrt{x-3}$.

a) Obtain an expression for $\frac{dy}{dx}$ in the form $\frac{ax+b}{2\sqrt{x-3}}$, where *a* and *b* are constants.

b) Determine the values of *x* for which *y* is an increasing function.

c) The variables *x* and *y* are such that, when *x* = 4, *y* is increasing at a rate of 0.2 units per second.

Find the rate of change of *x* when *x* = 4.

d) It is given further that variable $z=2y^3$.

Show that, when *x* = 4, *z* is increasing at 1734 times the rate of *y*.

Question 25:

A curve has the equation $y=\frac{\ln x}{x^2}$.

a) State the range of values of *x* for which the curve will be valid.

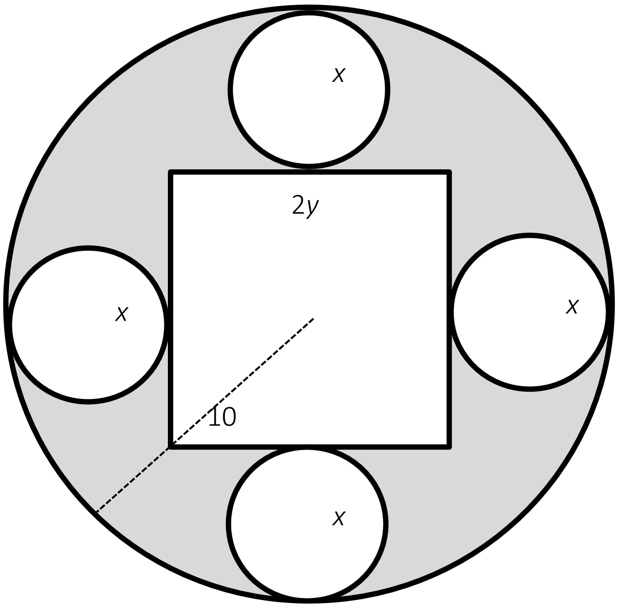
b) Find the *x* coordinate of the stationary point.

c) Determine the nature of the stationary point.

Question 26:

Differentiate $\frac{x^4}{\sqrt{x^2+1}}$ with respect to *x*.

Question 27:



The owner of a circular plot of land of radius 10m is planning to lay a central square lawn of side 2*y* m and four non-overlapping circular flower beds, each of radius *x* m, as shown in the diagram. The beds touch the lawn at the mid-points of its sides, and touch the circular edge of the plot.

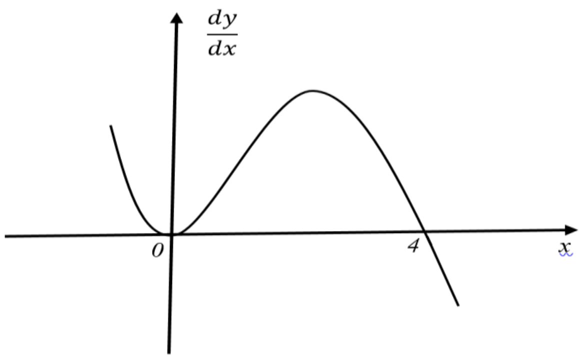
a) Express *y* in terms of *x*.

b) If *S* is the total area of the lawn and the flower beds, show that $S = (16 + 4\pi)x^2 - 160x + 400$.

c) Using differentiation, determine the least value of *S*, giving your answer in terms of π.

Question 28:

The diagram below shows the graph of the gradient function $\frac{dy}{dx}$ against *x* for the curve y = f(*x*) . The graph has axial intercepts at (0, 0) and (4, 0).



a) Find the range of *x* for which the curve is a decreasing function.

b) State the *x*-coordinates of the stationary points of the curve of *y* = f(*x*) and its nature stating reasons clearly

Question 29:

It is given that $x = e^{-2t} + 3e^{-t}$.

a) Prove that $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 3\frac{\mathrm{d}x}{\mathrm{d}t} + 2x = 0$.

b) Write down an expression for $\frac{d^3x}{dt^3}$ in terms of *t* and hence, find an expression for $\frac{\mathrm{d}^nx}{\mathrm{d}t^n}$ in terms of *n* and *t*.

Question 30:

Find the equation of the tangent and normal to the curve $y = \frac{2x-6}{x-2}$ at the point where the curve crosses x axis.

Question 31:

The equation of a curve is $y = e^{\frac{1}{2}x} - e^{-\frac{1}{8}x}$.

a) Show that $\frac{dy}{dx} = \frac{ae^{bx}+1}{ce^{\frac{1}{8}x}}$, where a, b and c are positive constants.

b) Explain why y is an increasing function for all real values of x.

Question 32:

Given that $y = \frac{\ln x}{2x-1}$, find

a) an expression for $\frac{dy}{dx}$.

b) the rate of change of x at the instant when $x = 6$, given that y changes at a rate of 5 units per second at this instant.

Question 33:

A curve has equation $y = x\cos^2 x$. The point P on the curve has x-coordinate $x = \frac{\pi}{4}$. The tangent and normal to the curve at point P intersects the y-axis at points Q and R respectively.

a) Show that the equation of the tangent at P is $y = \left(\frac{2-\pi}{4}\right)x + \frac{\pi^2}{16}$.

b) Find the area of triangle PQR.

Question 34:

A curve has the equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{x-5}{3x+1}$ for $x > 0$.

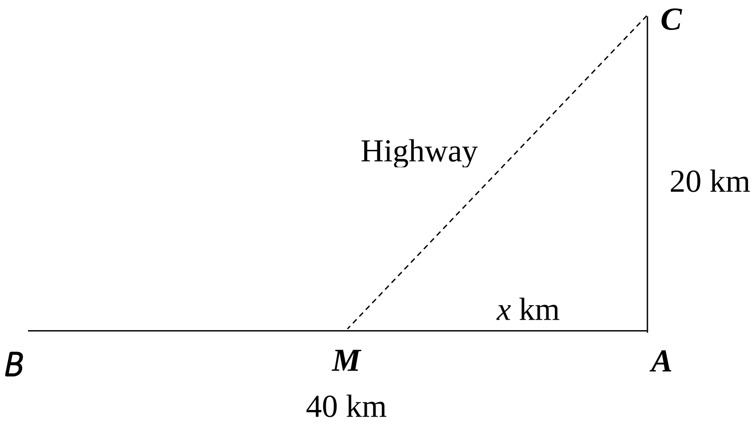
a) Obtain an expression for $\mathrm{f}'(x)$.

b) Hence explain whether $\mathrm{f}(x)$ is an increasing or decreasing function.

c) Showing full working, determine whether the gradient of the curve is an increasing or decreasing function.

Question 35:

The diagram below shows a straight road of 40 km long linking Town *A* and Town *B*. An industrial Town *C* is located 20 km due north of Town *A* and it is where Town *B* obtains many of its essential supplies. A new straight highway is to be built from Town *C* to a point *M* along the road *AB*. The distance of *M* from *A* is *x* km.



When the highway is built, transport trucks will move the supplies from Town *C* along the highway to point *M*, before continuing the remaining of the journey to Town *B* along road *AB*. The charges of using any stretch of road *AB* is \$2 per km, while that of using the new highway is \$5 per km.

a) Show that the total transportation cost \$*T* to move supplies from Town *C* to Town *B* after the highway is completed can be expressed as

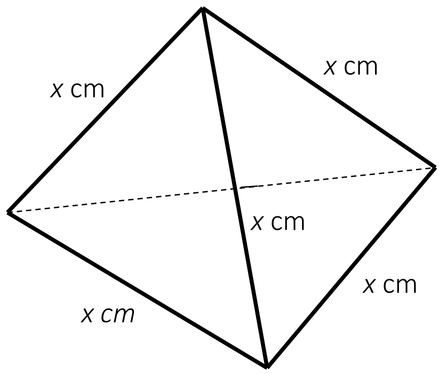
$$T = 80 - 2x + 5\sqrt{400 + x^2}$$

b) Determine the value of *x* for which *T* is stationary.

c) Show that this value of *x* makes *T* a minimum, and find this minimum value of *T*, correct to the nearest dollar.

Question 36:

The diagram shows a regular tetrahedron made up of four equilateral triangles. Each triangle has side *x* cm (diagram is not drawn to scale).



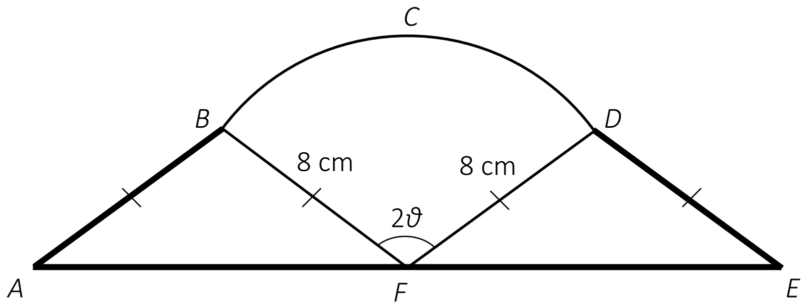
a) Show that the area of each triangle is $\frac{\sqrt{3}}{4}x^2\ \text{cm}^2$.

b) The tetrahedron is expanding such that the total surface area is increasing at a constant rate of 50 cm^2/s . At the instant when *x* = 10 cm, find the rate of increase of the side.

c) Given that the perpendicular height of this tetrahedron is $\frac{\sqrt{5}}{3}x$ cm. The formula for the volume of a pyramid is $V = \frac{1}{3} \times \text{base area} \times h$ where *h* is the perpendicular height.
At the instant when *x* = 10 cm, find the rate of increase of the volume.

Question 37:

The cross section of a helmet is in the shape of *ABCDEF*. *BCDF* is a sector of circle centre *F* of radius 8 cm, such that angle *BFD* = 2θ radians. Triangle *ABF* and triangle *FDE* are identical isosceles triangles as shown. *AFE* is a straight line.



a) Show that the area, *A* cm², of *ABCDEF* is

$$A = 64\theta + 64\sin 2\theta.$$

b) Given that ϑ can vary, find the value of ϑ for which the area of the cross section is a maximum.

Question 38:

The equation of a curve is $y = x^3 - 7x^2 - 5x + m$, where m is a constant.

a) Find the set of values x for which y is decreasing.

b) Find the possible values of m for which the x–axis is a tangent to the curve.

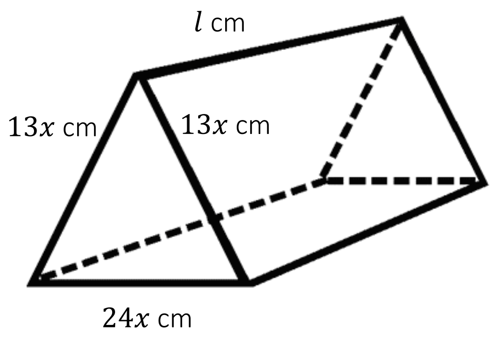
Question 39:

By differentiating $\frac{1}{\cos(ax)}$, where a is a constant, show that $\frac{d}{dx}[sec(ax)] = atan(ax)sec(ax)$.

39b) Given the curve $y = x^2 - \sec 6x$, find the exact value of $\frac{dy}{dx}$ when $x = \frac{\pi}{8}$.

If y increases at a constant rate of 2.5 radians per second, find the rate of change of x when $x = \frac{\pi}{8}$.

Question 40:



The diagram shows a triangular prism whose cross-section is an isosceles triangle with sides $13x$ cm, $13x$ cm and $24x$ cm. The length of the prism is l cm and its volume is 500 cm^3.

a) Show that the total surface area of the prism, A cm^2, is given by

$$A = 120x^2 + \frac{1250}{3x}$$

b) Given that x can vary, find the stationary value of A and determine whether this stationary value is a maximum or a minimum.