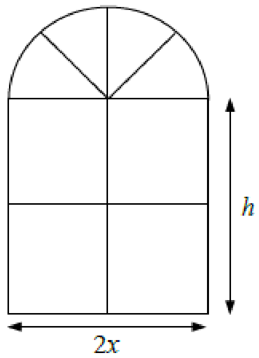
Equity Learning Place

Additional Math Topical (Differentiation I)

Question 1:

A particle moves along the curve $y = 3 - \frac{2}{x^2}$ in such a way that the y-coordinate of the particle is increasing at a constant rate of 0.02 units per second. Find the y-coordinate of the particle at the instant that the x-coordinate of the particle is increasing at a rate of 0.32 units per second.

Question 2:



The diagram shows a window in the shape of an adjoining semicircle and a rectangle of width $2x$ m and heigh h m. The perimeter of the window is 8 m

i) Show that the area, $A\ \mathrm{m}^2$, of the window is given by $A = 8x - \left(\frac{\pi+4}{2}\right)x^2$

ii) Given that x can vary, find the value of x for which A is a maximum.

iii) Find the maximum value of A

Question 3:

The point A lies on the curve $y = 2x \ln 2x$. The tangent to the curve at A is parallel to the line $8y = 32x - 3$.

i) Find the coordinates of A

The normal to the curve $y = 2x \ln 2x$ at A meets the line $8y = 32x - 3$ at the point B.

ii) Show that the $x - \text{coordinate}$ of B is $k(1 + 3e)$, where k is a constant to be found.

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Question 4:

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Question 5:

The equation of a curve is $y = 4(2-3x)^5 - 1$. The point $(a\,,b)$ is the stationary point on the curve.

i) Find the value of a and of b.

ii) Explain whether y is increasing or decreasing for $x \neq a$

iii) Using your result in (ii), deduce the nature of the stationary point.

iv) find the value of $\frac{d^2y}{dx^2}$ at the stationary point.

Question 6:

A curve has the equation $y = \dfrac{x^2+3}{x-1}$, $x \neq 1$. Find the coordinates of the stationary point(s) of the curve and determine the nature of the stationary point(s).

Question 7:

Given that $y = \sec^2 2x$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

Question 8:

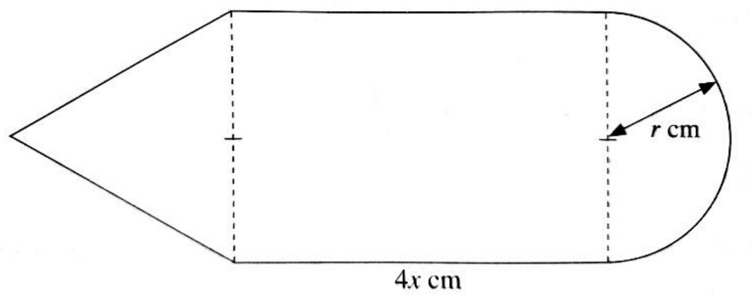
Let the function $y = e^{2x}\left(x^2 - x + 1\right)$.

a) Explain why the function is increasing for all values of x.

b) If $\dfrac{d^2y}{dx^2} - m\dfrac{dy}{dx} + 4y = 2e^{2x}$, find the value of m.

Question 9:

A piece of wire 80 cm long is bent to form a shape as shown in the diagram. The shape is made up of a semi-circular arc of radius r cm and two sides of an equilateral triangle on the opposite ends of a rectangle whose length is $4x$ cm.
The enclosed area of the shape is A cm^2.



a) Express x in terms of r.

b) Show that $A = 80r + r^2\left(\sqrt{3} - 4 - \frac{\pi}{2}\right)$.

c) Find the stationary value of A and determine if this value of A is a maximum or a minimum.

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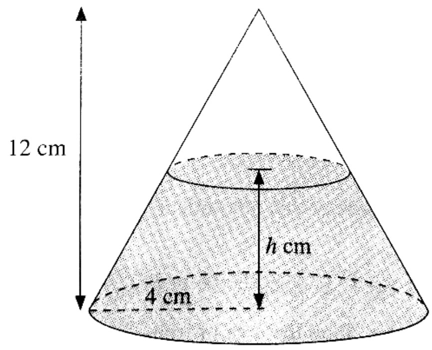
Question 10:

The diagram shows a conical container, initially full, of height 12 cm and radius 4 cm. Water is being pumped out of the container through an opening at the base of the container at a constant rate of 8 cm^3/s. After t seconds, the depth of the water is h cm.



a) Show that the volume of the water in the container, V cm^3, at time t seconds is given by

$$V = 64\pi - \frac{\pi}{27}(12-h)^3$$

b) Find the rate of change of the depth when the volume of water in the container is 37π cm^3.

c) A person claims that as more time passes, the rate of change of the depth of the water will decrease. Is the person correct? Justify your answer.

Question 11:

Express $\frac{4x-7}{(x-3)(x^2+1)}$ in partial fractions.

Hence, find the gradient at the point where the curve $y = \frac{4x-7}{(x-3)(x^2+1)}$ cuts the *y*-axis.

Question 12:

An ice cube retains its shape as it melts. The length, *l* cm, of each edge decreases at a constant rate of 0.04 cm/s. Find

a) the time taken for the surface area to decrease from 720 cm^2 to 240 cm^2.

b) the rate of change of the volume when the length of the edge is 15 cm.

Question 13:

Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\cos x}{1-\sin x}\right)$ can be written in the form $\frac{k}{1-\sin x}$.

Hence state the value of k.

Question 14:

Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left[(3x+2)\sqrt{4x+1}\right]$ can be expressed as $\frac{mx+n}{\sqrt{4x+1}}$ where *m* and *n* are constants to be determined. State the value of *m* and *n*.

Question 15:

A curve has the equation $y = \frac{4x^2 - 5}{2x - 1}$.

a) Find the equation of the tangent to the curve at the point (1, –1).

b) Show that this tangent does not meet the curve again.

Question 16:

A curve has the equation $y = \frac{\sin 2x}{e^{2x+1}}$.

a) Show that *y* is an increasing function of *x* for $0 < x < \frac{\pi}{8}$.

b) Find the exact value of the gradient of the curve when *x* = 0.

Question 17:

Differentiate $\ln\left(\frac{1+x}{x+3}\right)$ with respect to *x*.

a) Explain why the curve $y = \ln\left(\frac{1+x}{x+3}\right)$ has no stationary point, where $x \geq 0$.

b) Find the *x*-coordinate of the point at which the tangent to the curve

$y = \ln\left(\frac{1+x}{x+3}\right)$ is parallel to $4y - 12 = x$.

Question 18:

It is given that f $(x) = (x-2)(2x^2 - 5x + 7)$.

a) Explain why the equation $f(x) = 0$ has only one real root and state its value.

b) Find the value of the constant *k* for which the graph of $y = f(x) + kx$ has a stationary point at which $\frac{d^2y}{dx^2} = 0$. Hence, determine the nature of

this stationary point.

Question 19:

The function f is defined, for all values of x, by $f(x) = -2x^3 + \frac{5}{2}x^2 + 4x - 5$. Find the values of x for which f is a decreasing function.

Question 20:

The equation of a curve is $y = e^{-3x}\, x^2$.

a) Find the coordinates of the stationary points of the curve.

b) Determine the nature of each of these points.