Additional Math Topical (Trigonometry (I))

Question 1:

i) Prove that $\frac{1-\cos 2A}{1+\cos 2A}=\tan^2 A$.

ii) Solve the equation $\frac{1-\cos 2A}{1+\cos 2A}=3\tan A$ for $0° < A < 360°$.

Question 2:

The function f is defined by $f(x)=2\cos 2x-1$.

i) State the amplitude of f.

ii) State the period of f.

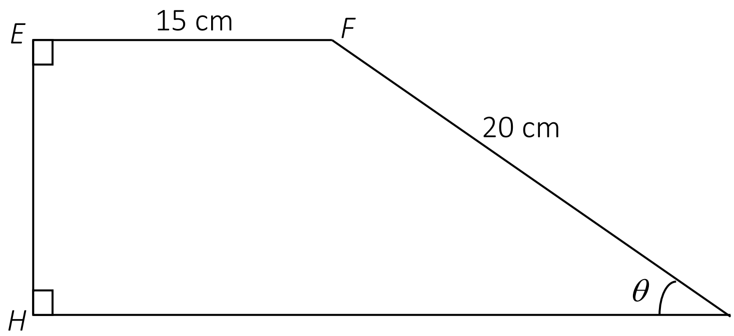
The equation of a curve is $y=2\cos 2x-1$ for $0 \le x \le 2\pi$.

iii) Sketch the graph of $y=2\cos 2x-1$ for $0 \le x \le 2\pi$.

iv) Hence, by drawing a suitable line on the same diagram, find the number of solutions for the equation $2\cos 2x+x=\pi+1$ where $0 \le x \le 2\pi$

Question 3:

The diagram shows a trapezium *EFGH* where *EF* = 15 cm and *FG* = 20 cm. Angle *FGH* = θ° and angle *FEH* = angle *EHG* = 90°.



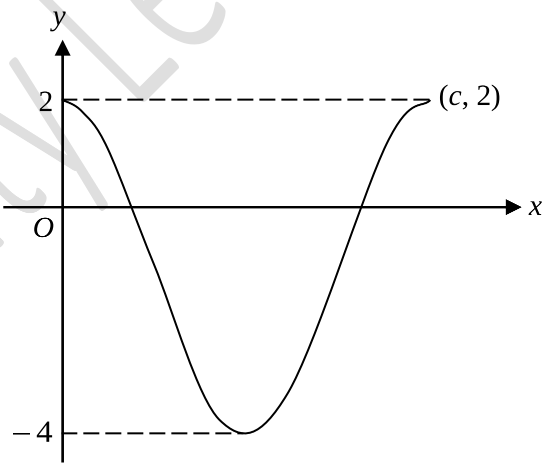
i) Explain clearly why the perimeter, *P* cm, of trapezium *EFGH* is *P* = 50 + 20 sin θ + 20 cos θ.

ii) Express *P* in the form $S + R\sin(\theta + \alpha)$, where *S*, *R* and α are constants and 0° < α < 90°.

iii) Find the maximum value of *P* and the corresponding value of θ.

iv) Show that the area, *A* cm^2, of trapezium *EFGH* is *A* = 300 sin θ + 100 sin 2θ.

Question 4:



The diagram shows part of the graph of *y* = *a* cos 2*x* + *b* where *x* is in radians. Find the value of each of the constants *a*, *b* and *c*.

Additional Math Topical (Trigonometry (I))

Question 5:

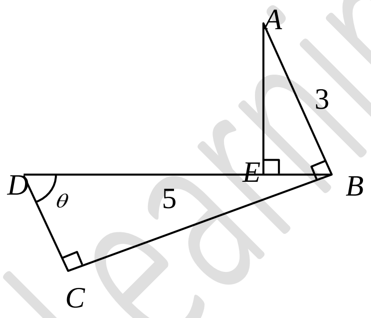
i) Prove that $\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$.

ii) Hence solve the equation $\frac{\sin 2\theta}{1+\cos 2\theta} = 4$ for $0° \le \theta \le 360°$.

Question 6:

Solve the equation $\cot\left(2x - \frac{\pi}{6}\right) = 2$ for $0 < x < 2\pi$.

Question 7:



The diagram shows two right-angled triangles ABE and BCD where $AB = 3$ cm, $BD = 5$ cm and $\angle ABC = \angle AEB = \angle BCD = 90°$. The angle θ is a variable angle where $0° < \theta < 90°$.

i) Show that $AE + ED + DC + CB = (5 + 2\cos\theta + 8\sin\theta)$.

ii) Express $AE + ED + DC + CB = p + R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle.

iii) State the maximum value of $AE + ED + DC + CB$ and the corresponding value of θ.

Question 8:

Express $\frac{\sin 60° - \cos 60°}{\cot 30° + \tan 45°}$ in the form $a\sqrt{3} + b$.

Question 9:

Given that $\sin\beta = k$ where β is less than $\frac{\pi}{2}$, express the following in terms of *k*.

a) $\cos\beta$

b) $\sin(\pi+\beta)$

c) $\cot\left(\frac{\pi}{2}-\beta\right)$

Question 10:

Find all the angles between 0° and 360° which satisfy the following equations.

a) $8\sin y\cos y+\sin^2 y=0$

b) $\tan(2x-30°)=2$

Question 11:

Prove the identity $\frac{(1-\cos x)^2+\sin^2 x}{\sin x-\sin x\cos x}=2\operatorname{cosec} x$.

Hence, solve the equation $\frac{(1-\cos x)^2+\sin^2 x}{\sin x-\sin x\cos x}-4=0$, for $0° \le x \le 360°$.

Question 12:

Sketch the graph of $y = 3\cos 2x + 5$ for $0 \le \theta \le 180°$

Question 13:

Prove that $(1+\sin\theta)(\sec\theta-\tan\theta)=\cos\theta$.

Question 14:

Given $\cos\theta = -\frac{2}{3}$ and that $\cos\theta$ and $\tan\theta$ have the same sign. Evaluate, without using a calculator

i) $\sin\theta$

ii) $\sin\left(\frac{\pi}{2} - \theta\right)$

iii) $\tan\theta$

iv) $\cos(\pi - \theta)$

Question 15:

The function *f* is defined by $f(x) = 2\sin x - 1$ for $0 \le x \le 2\pi$.

a) State the amplitude and period of *f*.

b) Sketch the graph of $y = f(x)$, stating the coordinates of the maximum and minimum points.

Question 16:

Solve each of the following equations

a) $2\cos(y + 45°) = -1$ for $0° \le y \le 360°$,

b) $\left(\sin x - \frac{1}{2}\right)(1 - 3\tan x) = 0$ for $0 \le x \le 2\pi$,

c) $4(\sin x + 1) = \cos 2x + 9$ for $0 \le x \le 2\pi$, leaving your answers in terms of π.

Question 17:

Given that $\cos\theta = -\frac{4}{5}$ and $180^\circ < \theta < 270^\circ$, find the exact value of

a) $\tan\theta$,

b) $\sin(-\theta)$,

c) $\cos\frac{\theta}{2}$

Question 18:

a) State the values, in degrees, between which the principal value of $\tan^{-1}x$ must lie.

b) On the same diagram sketch the graphs of $y = 3\sin x$ and $y = \cos x - 1$ for $0° \le x \le 360°$.

Hence, state the number of solutions for $3\sin x + 1 = \cos x$.

c) Prove that tan *A* + cot *A* = sec *A* cosec *A*.

Question 19:

a) Express $\tan 45°$ and $\tan 30°$ in its exact form.

Hence, show, without using a calculator, that $\tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1}$.

b) Find the values of ϑ for $\cos 2\theta = 0.4$ such that $0 < \theta < 3$ radians.

c) Find the values of *x* for $16\sin^2 x = 14 - 4\cos x$ such that $0° < x < 360°$.