# AM Topical Question

Further Coordinate Geometry

1) A circle $C_1$ whose equation is $x^2 + y^2 - 6x - 4y - 12 = 0$, has centre $A$ and radius $r$.

i) Find the coordinates of $A$ and the value of $r$.

ii) Show that the point $B(6,-2)$ lies on the circle $C_1$.

iii) Find the equation of the perpendicular bisector of $AB$.

iv) The perpendicular bisector of $AB$ is the tangent to another circle $C_2$, with centre $B$. Find the equation of circle $C_2$.

(i) $x^2 - 6x + y^2 - 4y - 12 = 0$

$(x-3)^2 - 9 + (y-2)^2 - 4 - 12 = 0$

$(x-3)^2 + (y-2)^2 = 25$

$\therefore A(3,2)$ and $r = 5$

(ii) Dist $AB = \sqrt{(6-3)^2 + (-2-2)^2}$

$= \sqrt{9+16}$

$= 5$

Since $AB$ = radius, $B$ lies on $C_1$

(iii) Grad $AB = \dfrac{-2-2}{6-3} = -2$

Grad $\perp$ bisector $= \frac{1}{2}$

mid pt $AB = \left(\frac{6+3}{2}, \frac{-2+2}{2}\right)$

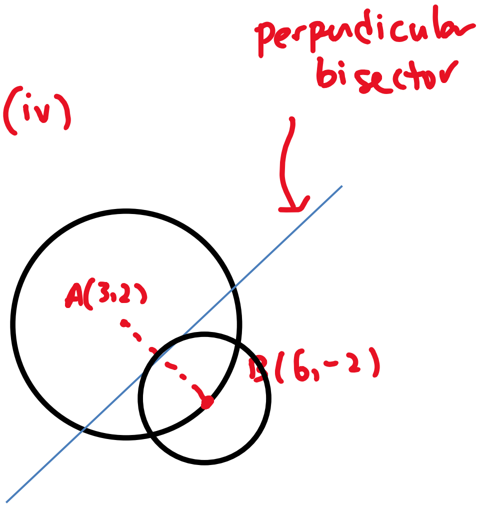
$= \left(\frac{9}{2}, 0\right)$

$0 = \frac{1}{2}\left(\frac{9}{2}\right) + c$

$c = -\frac{9}{4}$

Eqn : $y = \frac{1}{2}x - \frac{9}{4}$

(iv)




The other circle radius is $\frac{5}{2}$ cm.

$(x-6)^2 + (y+2)^2 = \frac{25}{4}$

2) A circle $C_1$ has equation given by $x^2 + y^2 - 2x + 6y - 15 = 0$ and the point $P(4,-7)$ is on $C_1$.

i) Find the radius and the coordinates of the centre of $C_1$.

ii) The point $Q$ is such that $PQ$ is the diameter of the circle. Find the coordinates of $Q$.

iii) A new circle $C_2$ passes through $P$, $Q$ and $R$, where $R$ is a point outside the circle $C_1$ such that angle $PRQ = 45°$. Explain briefly if it is possible for the centre of $C_2$ to lie on the circumference of $C_1$.

i) $x^2+y^2-2x+6y-15=0$

$x^2-2x+y^2+6y-15=0$

$(x-1)^2-1+(y+3)^2-9=15$

$(x-1)^2+(y+3)^2=25$

$\therefore$ centre $(1,-3)$ and radius $=5$

ii) Let $Q(x,y)$

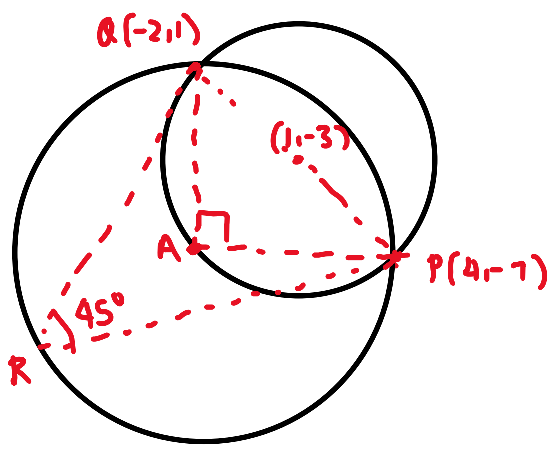
$\left(\frac{4+x}{2}, \frac{y-7}{2}\right)=(1,-3)$

$\frac{4+x}{2}=1$ , $\frac{y-7}{2}=-3$

$x=-2$ , $y=1$

$\therefore Q(-2,1)$

iii)




Yes, the centre of circle must lie on $C_1$ because by the property angle at centre equal twice angle at circumference,

$QAP = 90°$

By converse of right ∠ in semi-circle property, A must be on the circumference of $C_1$.

3) The equation of a circle, $C_1$ with centre *A*, is $x^2 + y^2 + 6x - 4y - 7 = 0$.

a) Find the coordinates of $A$ and the radius of the circle.

b) Circle $C_1$ intersects the *x*-axis at *P* and *Q*. Find the coordinates of *P* and of *Q*.

c) A second circle, $C_2$ with centre *B*, also passes through *P* and *Q*. State the *x*-coordinate of *B*.

d) Given that the *y*-coordinate of *B* is negative and that the radius of $C_2$ is $\sqrt{41}$, find the *y*-coordinate of *B*.

e) Write down the equation of circle $C_2$.

a) $x^2 + 6x + y^2 - 4y - 7 = 0$

$(x+3)^2 - 9 + (y-2)^2 - 4 - 7 = 0$

$(x+3)^2 + (y-2)^2 = 20$

$A(-3, 2)$ radius $= \sqrt{20}$

b) $x^2 + y^2 + 6x - 4y - 7 = 0$

Let $y = 0$,

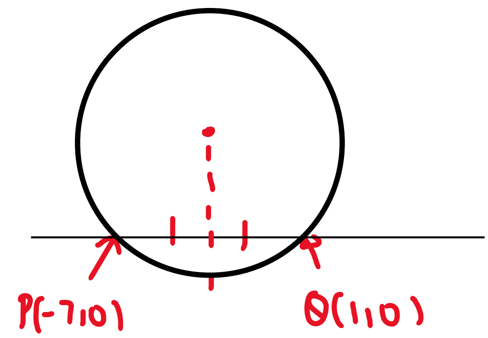
$x^2 + 6x - 7 = 0$

$(x+7)(x-1) = 0$

$x = -7$ or $x = 1$

$\therefore P(-7, 0)$ and $Q(1, 0)$

c)




x coordinate of centre $= \dfrac{-7+1}{2}$

$= -3$

d) Let centre of $C_2$ be $(-3, y)$

$\sqrt{41} = \sqrt{(-3+7)^2 - (y)^2}$

$41 = 16 + y^2$

$y^2 = 25$

$y = \pm 5$ (rej $+5$)

$\therefore$ y-coordinate is $-5$.

e) Eqn. of $C_2$ :

$(x+3)^2 + (y+5)^2 = 41$

4) The negative $x$-axis and the line $y = 10$ are tangents to a circle $C$.

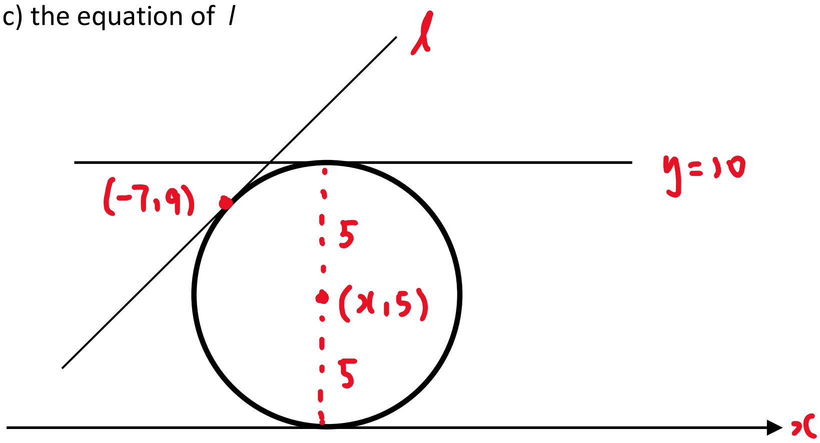
a) What can be deduced about the coordinates of the centre of $C$ and the relationship between the radius of $C$ and the $y$-coordinate of the centre of $C$?

The line $l$ is a tangent to $C$ at the point $P$. The point $P$ has coordinates $(-7, 9)$. Given that the centre of $C$ lies below and to the right of $P$, find

b) the equation of $C$

c) the equation of $l$

4)




a) The x-coordinate is negative.
The y-coordinate is 5.
The radius is 5

b) $5 = \sqrt{(x+7)^2+(5-9)^2}$

$25 = (x+7)^2 + 16$

$(x+7)^2 = 9$

$x = -7+3$ or $-7-3$

$x = -4$ or $-10$ (N/A)

$\therefore$ Eqn. of C :

$(x+4)^2+(y-5)^2 = 25$

(c) Gradient of radius

$= \dfrac{9-5}{-7+4}$

$= \dfrac{4}{-3}$

Grad of $l$ $= \dfrac{3}{4}$

$9 = -7\left(\dfrac{3}{4}\right)+c$

$c = \dfrac{57}{4}$

$\therefore$ Eqn of $l$ :

$y = \dfrac{3}{4}x + \dfrac{57}{4}$

5) A circle, centre $C$, has a diameter $PQ$ where $P$ is the point $(-4, -3)$ and $Q$ is the point $(8, 5)$.

a) Find the coordinates of $C$ and the radius of the circle.

b) Find the equation of the circle.

c) Determine whether the point $R(-3.25, 5.5)$ lies inside, outside or on the circle. Show your working clearly.

a) midpt PQ $= \left(\frac{8-4}{2}, \frac{5-3}{2}\right)$

$C = (2,1)$

Radius $= \sqrt{(2+4)^2+(1+3)^2}$

$= \sqrt{50}$

b) $(x-2)^2+(y-1)^2 = 50$

c) dist CR $= \sqrt{(-3.25-2)^2+(5.5-1)^2}$

$= \sqrt{47.8125}$

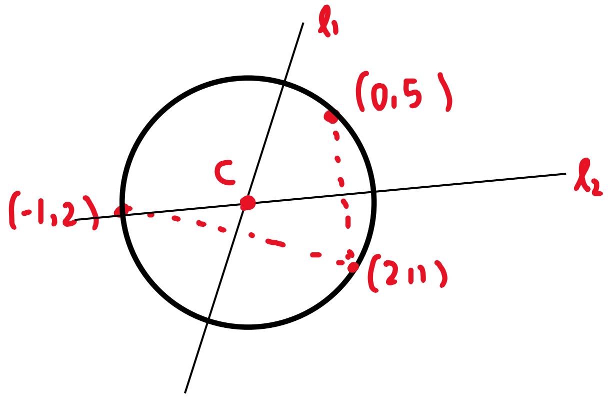
Since $CR < \sqrt{50}$, R is inside the circle.

6) A circle $C$ passes through the points $(-1, 2)$, $(0, 5)$ and $(2, 1)$.

a) Show that the coordinates of the centre of circle $C$ is $(1, 3)$.

b) Hence, find the equation of the circle $C$.

c) Find the equation of the circle which is a reflection of circle $C$ in the line $x = 3$.




$$\text{grad} = \frac{2-1}{-1-2} = \frac{1}{-3}$$

$$\text{grad of } l_1 = 3$$

$$\text{midpt} = \left(\frac{2-1}{2}, \frac{1+2}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$$

Eqn $l_1$:

$$\frac{3}{2} = 3\left(\frac{1}{2}\right) + c$$

$$c = 0$$

$$\therefore y = 3x \quad -(1)$$

$$\text{grad} = \frac{5-1}{0-2} = -2$$

$$\text{grad } l_2 = \frac{1}{2}$$

$$\text{mid pt} = \left(\frac{0+2}{2}, \frac{5+1}{2}\right) = (1, 3)$$

Eqn $l_2$:

$$3 = (1)\left(\frac{1}{2}\right) + c$$

$$c = \frac{5}{2}$$

$$\therefore y = \frac{1}{2}x + \frac{5}{2} \quad -(2)$$

Sub (1) into (2),

$$3x = \frac{1}{2}x + \frac{5}{2}$$

$$x = 1$$

$$y = 3$$

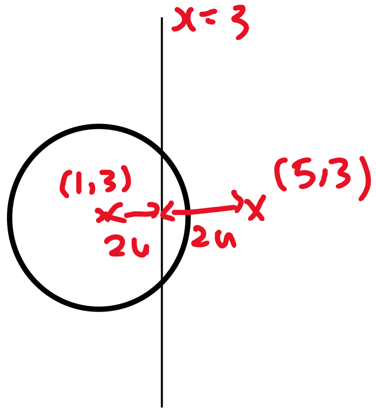
$$C(1,3)$$

b) $\text{Radius} = \sqrt{(1-2)^2 + (3-1)^2} = \sqrt{5}$

$\therefore$ eqn of circle:

$$(x-1)^2 + (y-3)^2 = 5$$

c)




Eqn new circle

$$(x-5)^2 + (y-3)^2 = 5$$

7) A circle, whose equation $x^2 + y^2 - 4x - 10y - 7 = 0$, has centre $C$ and radius $r$.

a) Find the coordinates of $C$ and the value of $r$.

$AB$ is a diameter of the circle and is parallel to the $y$-axis.

b) Find the coordinates of $A$ and $B$.

a) $x^2 - 4x + y^2 - 10y - 7 = 0$

$(x-2)^2 - 4 + (y-5)^2 - 25 - 7 = 0$

$(x-2)^2 + (y-5)^2 = 36$

$C\ (2, 5)$ and radius $= 6$

b) $A(2, -1)$ , $B(2, 11)$

8) The straight line $y = 5 - 2x$ meets the circle $x^2 + y^2 + 4x - 8y - 5 = 0$ with centre $C$ at two points $P$ and $Q$.

a) Find the coordinates of $C$ and the radius of the circle.

b) Find the general form of the equation of another circle for which $PQ$ is a diameter.

a) $x^2 + 4x + y^2 - 8y - 5 = 0$

$(x+2)^2 - 4 + (y-4)^2 - 16 - 5 = 0$

$(x+2)^2 + (y-4)^2 = 25 \quad - (2)$

$C(-2, 4)$ and radius $= 5$

b) $y = 5 - 2x \quad - (1)$

Sub (1) into (2),

$(x+2)^2 + (1-2x)^2 = 25$

$x^2 + 4x + 4 + 4x^2 - 4x + 1 = 25$

$5x^2 - 20 = 0$

$x^2 = 4$

$x = \pm 2$

$y = 1$ or $y = 9$

$\therefore P(2, 1)$, $Q = (-2, 9)$

midpt $PQ = \left( \frac{2-2}{6}, \frac{1+9}{2} \right)$

$= (0, 5)$

Length $PQ = \sqrt{(2+2)^2 + (1-9)^2}$

$= \sqrt{80}$

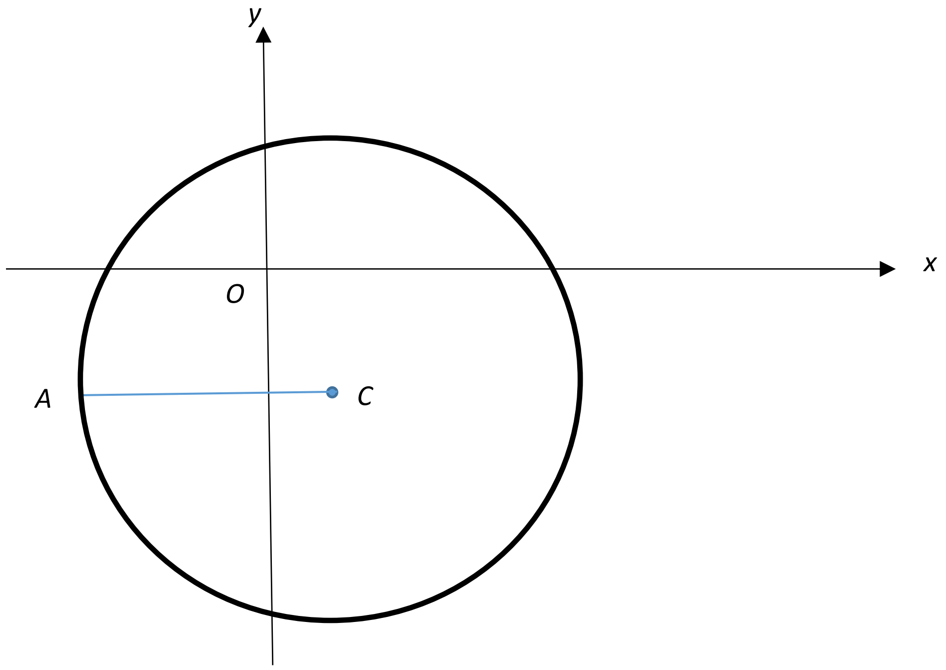
Radius $= \frac{\sqrt{80}}{2} = \sqrt{20}$

Eqn: $x^2 + (y-5)^2 = 20$

$x^2 + y^2 - 10y + 25 = 20$

$x^2 + y^2 - 10y + 5 = 0$

9) The diagram below shows a circle with center $C\,(2,-1)$ and radius 5.




(i) Given that the equation of the circle is $x^2+y^2+2gx+2fy+c=0$, find the value of each of the constants $g$, $f$ and $c$.

(ii) The point $A$ lies on the circle such that $AC$ is parallel to the $x$-axis, write down the coordinates of $A$.

(iii) The points $B$ and $D$ lie on the circle and forms the diameter. If the coordinates of $B$ are (6,2), find the coordinates of $D$.

(i) $(x-2)^2+(y+1)^2=25$

$x^2-4x+4+y^2+2y+1=25$

$x^2+y^2-4x+2y-20=0$

$\therefore\ g=-2,\ f=1,\ c=-20$

(ii) $A(-3,-1)$

(iii) Let $D(x,y)$

$\left(\frac{x+6}{2}, \frac{y+2}{2}\right)=(2,-1)$

$\frac{x+6}{2}=2$, $\frac{y+2}{2}=-1$

$x=-2$, $y=-4$

$\therefore D(-2,-4)$

10) *C* is a circle with equation $2x^2 + 2y^2 + 2x - y = 5$.

a) Find the centre and radius of *C*.

b) If a line of equation $y = mx + c$ is parallel to $y + 2x = 1$, what can one conclude about the value of *m*?

c) There are two lines parallel to $y + 2x = 1$ which are tangent to *C*. Find the equation of these two lines.

a) $2x^2+2y^2+2x-y=5$

$x^2+y^2+x-\frac{1}{2}y=\frac{5}{2}$

$x^2+x+y^2-\frac{1}{2}y=\frac{5}{2}$

$(x+\frac{1}{2})^2-\frac{1}{4}+(y-\frac{1}{4})^2-\frac{1}{16}=\frac{5}{2}$

$(x+\frac{1}{2})^2+(y-\frac{1}{4})^2=\frac{45}{16}$

Centre $(-\frac{1}{2},\frac{1}{4})$ radius $=\frac{\sqrt{45}}{\sqrt{16}}$

$=\frac{3\sqrt{5}}{4}$

b) $y=-2x+1$,

$m=-2$.

c) $y=-2x+c$ — ①

$2x^2+2y^2+2x-y=5$ — ②

Sub ① into ②.

$2x^2+2(-2x+c)^2+2x-(-2x+c)=5$

$2x^2+2(4x^2-4xc+c^2)+4x-c-5=0$

$10x^2-8xc+4x+2c^2-c-5=0$

Since ① is tangent to ②,

$b^2-4ac=0$

$(-8c+4)^2-4(10)(2c^2-c-5)=0$

$64c^2-64c+16-80c^2+40c+200=0$

$16c^2+24c-216=0$

$2c^2+3c-27=0$

$(2c+9)(c-3)=0$

$c=-\frac{9}{2}$ or $c=3$

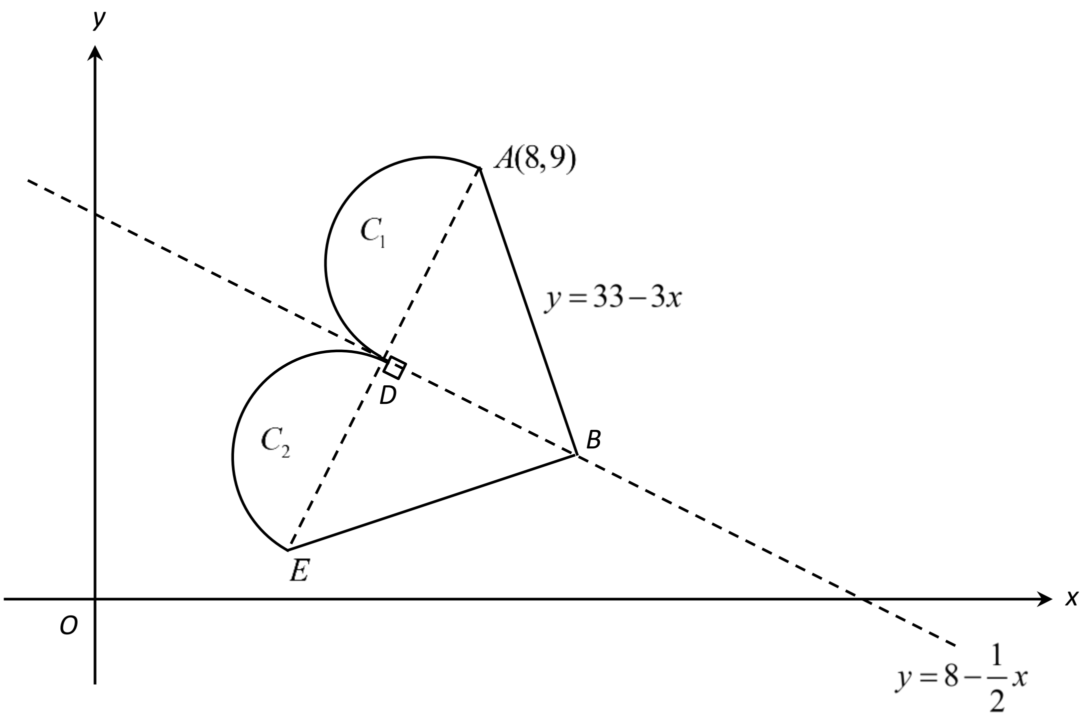
Eqn of lines:

$y=-2x+3$

$y=-2x-\frac{9}{2}$

**11** **Solutions to this question by accurate drawing will not be accepted.**

Mandy drew a heart-shaped figure as shown in the diagram below.




She started by drawing the line $y = 33 - 3x,$ which passes through points $A(8,9)$ and $B$. She then drew the line of symmetry $y = 8 - \frac{1}{2}x$ through *B* and a dotted line through *A* which is perpendicular to the line of symmetry. The point of intersection between the two lines is *D*.

**(i)** Find the coordinates of *B*. [2]

**(ii)** Show that the coordinates of *D* are (6, 5). [4]

She then plotted point *E*, which is the image of $A(8,9)$ under reflection about the line of symmetry. Finally, Mandy drew two circles $C_1$ and $C_2$ with diameters *AD* and *DE* respectively, before erasing half of the circles to form two semicircles.

**(iii)** Find the equation of the circle $C_1$ which Mandy initially drew. [4]

**(iv)** Find the equation of the line *BE* and use this equation to verify that the point *F*(7, 2) lies on *BE*. [4]

**(v)** To complete her drawing, Mandy drew an arrow through the points $A(8,9)$ and *F*(7, 2). Find the area of triangle *ABF*. [2]

11i) $y = 33 - 3x$ — ①

$y = 8 - \frac{1}{2}x$ — ②

sub ① into ②,

$33 - 3x = 8 - \frac{1}{2}x$

$25 = \frac{5}{2}x$

$x = 10$

$y = 3$

$\therefore B(10, 3)$

11ii) Grad AD $= 2$

Eqn AD :

$9 = 8(2) + c$

$c = -7$

$\therefore y = 2x - 7$ — ③

Sub ③ into ②,

$2x - 7 = 8 - \frac{1}{2}x$

$\frac{5}{2}x = 15$

$x = 6$

$y = 5$

$\therefore D(6, 5)$

11iii) midpt AD $= \left(\frac{8+6}{2}, \frac{9+5}{2}\right)$

$= (7, 7)$

Length AD $= \sqrt{(8-6)^2 + (9-5)^2}$

$= \sqrt{20}$

radius $= \frac{\sqrt{20}}{2} = \sqrt{5}$

Eqn $C_1$ :

$(x-7)^2 + (y-7)^2 = 5$

11iv) Let $E(x, y)$

$\left(\frac{x+8}{2}, \frac{y+9}{2}\right) = (6, 5)$

$\frac{x+8}{2} = 6$ , $\frac{y+9}{2} = 5$

$x = 4$ $y = 1$

$E(4, 1)$

Grad BE $= \frac{3-1}{10-4} = \frac{1}{3}$

Eqn BE :

$1 = \frac{1}{3}(4) + c$

$c = -\frac{1}{3}$

$\therefore y = \frac{1}{3}x - \frac{1}{3}$

Let $x = 7$,

$y = \frac{7}{3} - \frac{1}{3} = 2$

$\therefore F(7, 2)$ lies on BE

iv) Area ABF $= \frac{1}{2}\begin{vmatrix} 8 & 6 & 7 & 8 \\ 9 & 5 & 2 & 9 \end{vmatrix}$

$= \frac{1}{2}\left|(40+12+63)-(54+35+16)\right|$

$= 5 \text{ units}^2$

12) A circle $C_1$ has equation $x^2 + y^2 - 4x + 6y - 3 = 0$.

a) Find the coordinates of the centre and radius of circle $C_1$.

b) Find the coordinates on the circle $C_1$ where the gradient is equal to zero.

Another circle $C_2$ is a reflection about the line $y = 1$.

c) Find the equation of circle $C_2$.

12a) $x^2 - 4x + y^2 + 6y - 3 = 0$

$(x-2)^2 - 4 + (y+3)^2 - 9 - 3 = 0$

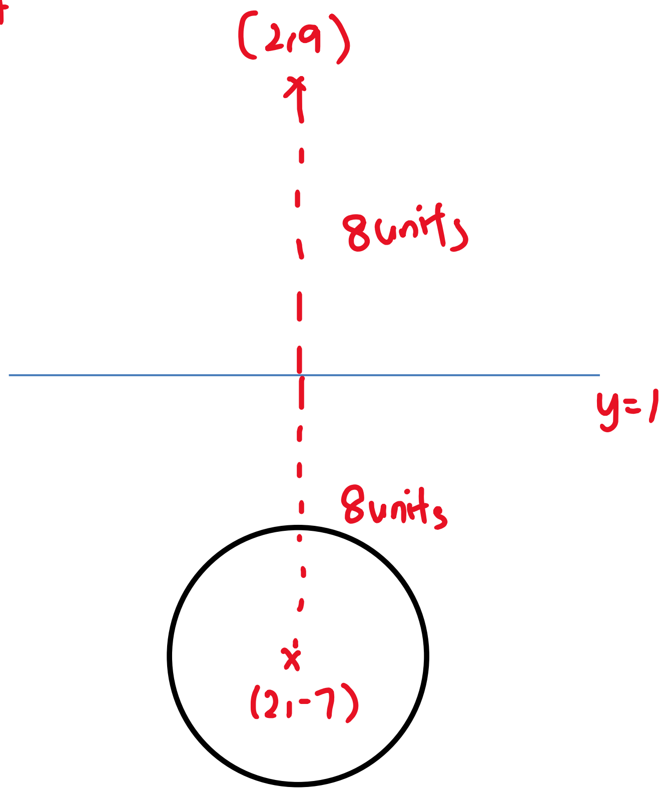
$(x-2)^2 + (y+3)^2 = 16$

centre $(2,-3)$, radius $= 4$

12b) $(2,-7)$ and $(2,1)$

12c) Eqn of $C_2$,

$(x-2)^2 + (y-9)^2 = 16$



13) The equation of a circle $C_1$, with radius 5 units, is given by $x^2 + y^2 + 6x - 10y + k = 0$.

a) Find the centre of $C_1$ and the value of $k$.

A second circle $C_2$ lies in the third quadrant of the Cartesian plane ($x < 0$, $y < 0$) and touches $C_1$ and the two axes.

b) Find the equation of circle $C_2$.

13a) $x^2 + 6x + y^2 - 10y + k = 0$

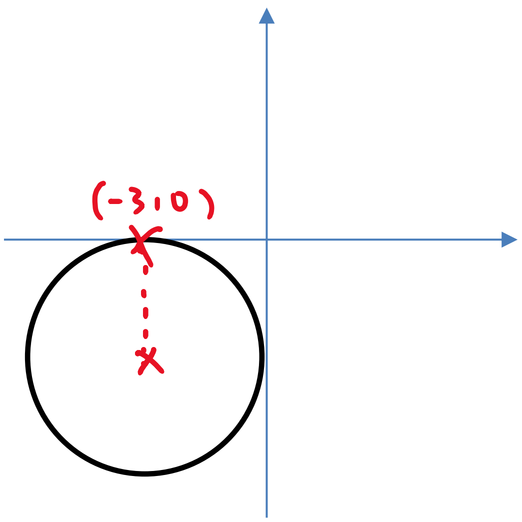
$(x+3)^2 - 9 + (y-5)^2 - 25 + k = 0$

$(x+3)^2 + (y-5)^2 = 34 - k$

centre $(-3, 5)$ and $34 - k = 25$

$k = 9$

13b)




$C_1$ touches $C_2$ at $(-3, 0)$

The radius of $C_2$ is 3

The y-coordinate of centre is -3

$\therefore (x+3)^2 + (y+3)^2 = 9$

14) The equation of a circle is $x^2 + y^2 - 10x - 4y + 25 = 0$.

a) Find the radius and the coordinates of the centre of the circle.

b) Show that the $x$-axis is tangent to the circle.

c) Find the equation of the circle which is a reflection of the original circle in the $x$-axis.

a) $x^2 - 10x + y^2 - 4y + 25 = 0$

$(x-5)^2 - 25 + (y-2)^2 - 4 + 25 = 0$

$(x-5)^2 + (y-2)^2 = 4$

radius $= 2$ , centre $(5,2)$

b) Let $y = 0$,

$x^2 - 10x + 25 = 0$

$(x-5)^2 = 0$

$x = 5$,

The circle cuts x-axis at one pt only. So it is tangent to the x-axis.

c)

$(x-5)^2 + (y+2)^2 = 4$

15) The equation of a circle $C_1$ is $x^2 + y^2 + 2x - 6y - 90 = 0$.

a) Find the centre and radius of the circle $C_1$.

b) The circle $C_1$ touches the line $y = a$ where $a > 0$. State the value of $a$.

c) The circle $C_1$ is reflected in the line $x = 10$. State the centre of this new circle.

d) A second circle $C_2$ has centre (2, 7) and a radius of 5 units. Justify with workings that the two circles $C_1$ and $C_2$ touch each other at only one point.

a) $x^2 + 2x + y^2 - 6y - 90 = 0$

$(x+1)^2 - 1 + (y-3)^2 - 9 - 90 = 0$

$(x+1)^2 + (y-3)^2 = 100$

centre $(-1, 3)$ radius $= 10$

b) $a = 13$

c) $(21, 3)$

d) $\sqrt{(2+1)^2 + (7-3)^2}$

$= 5$

Since the distance of the $C_1$ and $C_2$ is 5units

$C_2$ touches $C_1$ as illustrated at one point only.