EQUITY
LEARNING PLACE

Additional Math Topical (Logarithm I)

Question 1:

Given that $\log_2 x = p$ and $\log_4 y = q$, express the following in terms of *p* and/or *q*.

i) $\log_2 \sqrt{x}$,

ii) $\log_4 \frac{4x}{y}$,

iii) xy.

Question 2:

An object is heated to 50°C and then allowed to cool. Its temperature at time *t* minutes after it starts cooling is given by $T = T_0 + 16e^{\frac{-3}{4}t}$, where T_0 is a constant. Find

i) the value of T_0,

ii) the temperature of the object at 2 minutes,

iii) the time taken to reach 40°C,

iv) the approximate temperature when *t* becomes very large.

Question 3:

A certain radioactive substance is known to decay with time such that its mass, *M* grams, after *t* hours is given by $M = M_0 e^{-kt}$, where *k* is a positive constant. A block of this substance having a mass of 100 grams originally is observed. After 40 hours, its mass has decreased to 90 grams.

i) Find the mass of the substance after 100 hours.

ii) What will happen to the mass as *t* becomes very large? Explain your answer with clear working.

Question 4:

Given that $\log_4[\log_2(5k+6) - \log_2 3k] = \log_{25} 5$,

find, without the use of a calculator, the value of *k*.

Question 5:

Solve the following simultaneous equations

$$27^x \times \frac{1}{81^y} = \sqrt[4]{81}$$

$$\log_2 x - \log_4 y = 2$$

Question 6:

The price of a new car on 1 January 2017 is \$100 000. Given that the value of the car depreciates such that n months after the purchase, the sale price, \$$P$, is determined by the formula, $P = 100000 \times e^{-0.015n}$, estimate

a) the sale price of the car after 1 year, giving your answer correct to the nearest \$1000,

b) the month and year when the sale price of the car is less than \$50 000.

Question 7:

Solve the following equations

a) $\log_2(x^2 - 5) = 2 + \log_2 x$

b) $3\log_x 3 + \log_x 27 = \frac{3}{2}\log_3 x$

Question 8:

Given that $p = 3^x$ and $q = 3^y$, find in terms of x and/or y,

a) $\log_3 \frac{pq^2}{27}$.

b) $\log_3 3p - \log_9 q$

Additional Math Topical (Logarithm I)

Question 9:

A cup of hot chocolate was left to cool on a table so that t minutes later, its temperature is $T°C$ is given by $T = 40 + 35(0.63)^t$.

a) Find its initial temperature,

b) Find its temperature after 2 minutes,

c) Find the time, to the nearest minute, when its temperature is $62°C$,

d) State the value which T approaches as t becomes very large

e) Sketch the graph of $T = 40 + 35(0.63)^t$.

Question 10:

a) Given that $\log_2(2x-3) - \log_2(y+1) = 4$, express y in terms of x.

b) Solve $\log_5 x = 16\log_x 5$.

c) Solve the equation $3e^x - 2 = 4e^{-x}$.

Question 11:

The price, $\$P$, of a company share has been increasing each year. The company claims that this increase is exponential and can be modelled by an equation of the form

$$P = 8e^{kt},$$

where k is a constant and t is the time in years since the company was formed. Find

a) the initial value of the company share

b) the value of k if, after 5 years, the value of the company share has doubled,

c) Using the value of k in **(ii)**, the value of t when the value of the company share is 200% more than its original value.

Question 12:

Given that $\lg x = p$ and $\lg y = q$, express $\lg\left(x^y\right)^2$ in terms of p and q.

Additional Math Topical (Logarithm I)

Question 13:

a) Use the substitution $y = 3^x$ to express the equation $27^x + 3^{x+2} = 26$ as a cubic equation in *y*.

b) Show that $y = 2$ is the only real solution of this equation.

c) Hence solve the equation $27^x + 3^{x+2} = 26$.

Question 14:

Given that $p = \lg 3$ and $q = \lg 11$,

a) express $\lg\sqrt[3]{2970}$ in terms of *p* and *q*,

b) express *y* as a surd in the form $b\sqrt{33}$ where *b* is a constant such that $\lg y = \dfrac{3q - p}{2}$

Question 15:

Solve $\log_3(2x-17) = \log_9 81 - \log_3 x$.

Question 16:

A certain radioactive substance is known to decay with time such that its mass, *M* grams, after *t* hours is given by $M = M_o e^{-kt}$, where *k* is a positive constant. A block of this substance has an initial mass of 150 grams. After 30 hours, its mass has decreased to 90 grams.

a) Find the mass of the substance after 100 hours.

b) Find the value of *t* when the mass of the substance is one third its original value.

c) Explain why the mass of the substance can never reach zero gram.

Question 17:

Solve

a) $\log_{2x}(5x-1) = 2$

b) $2\log_2 y + \log_y 2 = 3 + y\log_y 1$

Question 18:

No Question

Question 19:

a) Simplify $\frac{1}{\log_a ab} + \frac{1}{\log_b ab}$.

b) Solve $\log_2(7 + x^2) = \log_2(5 - x) + 3$.

Question 20:

a) Solve the equation $2e^x - 10e^{-x} = 1$.

b) The population of a certain species of birds in a research study was found to be $P = P_o e^{kt}$, where P_o is original population of the birds at the start of the study, *t* is the time in years after the study and *k* is a constant.

i) Given that the population of the birds doubled after 10 years, show that the exact value of *k* is $\frac{\ln 2}{10}$.

ii) Calculate the original population of the birds if the population of the birds is 265 after 8 years.

Question 21:

Given that $\log_4 x = y$, without using a calculator, express

a) $\log_4 \sqrt{16x}$ in terms of y,

b) 2^y in terms of x.

Question 22:

Solve $\log_5\left[\log_p\left(2p^5 - 243\right)\right] = 1$.

Additional Math Topical (Logarithm I)

Question 23:

Given that $\log_9 m^4 = \log_{27} n^3$, express *m* in terms of *n*.

Question 24:

Solve

a) $\log_2(x+2) - \log_{\sqrt{2}}(x-1) = 1$,

b) $(3^{2x})(4^{x+2}) = e^2$.

Question 25:

The initial cancerous cell count *C* in an experiment was 72000. With the use of a new drug, the cancerous cell count decreases with time so that the cell count *C*, after *t* days is given by $C = 72000e^{ht}$, where *h* is a positive constant. The cancerous cell count drops to 18000 after 30 days.

a) Show that $h = -0.04621$ when rounded off to 5 decimal places.

b) Calculate the cancerous cell count *C* after 20 days, giving your answer correct to the nearest 100.

c) A drug is considered very effective if it can kill 90% of the cancerous cells in 50 days. Determine whether the new drug is very effective.

Question 26:

Solve the equation

a) $\lg(5+x) = \lg 5 + \lg x$,

b) $\log_2(x-1) = \log_4(x+5)$,

c) $2^{\frac{x}{2}} + 6 = 2^x$.

Additional Math Topical (Logarithm I)

Question 27:

The temperature, T °C, of a cup of hot water that has been left in a room is given by the formula $T = 24 + Ae^{-0.2t}$, where t is the time in minutes since the cup has been left in the room. It is given that when $t = 5,\ T = 50.$

a) Show that the value of A is 71, when estimated to a whole number.

b) Find the initial temperature of the water when it had just been left in the room.

c) Find the amount of time, in minutes and seconds, it takes for the initial temperature of the water to be halved.

d) Explain why the temperature of the water will never reach 20 °C.

Question 28:

A man buys a limited edition watch in January 2017. After t years, its value \$$V$ is given by $V = 20000 - 8000e^{-pt}$.

a) Find the value of the watch when the man bought it.

The value of the watch after 2 years is expected to be \$13,707. Calculate

b) the expected value of the watch after 3 years,

c) the year when the expected value of the watch will be \$18,000.

Question 29:

Solve the equation

a) $2\log_3(x+2) = 5 - \log_3 7$,

b) $\log_5 x = 2 + \log_x 125$.

Question 30:

At the beginning of the year 1990, the population of a certain species of birds was 15000. The population increased so that, after a period of n years, the new population was given by the equation $P = 15000(1.07)^n$. Find the year at which the population first doubled.