Additional Math Topical (Polynomial II)

Question 31:

Let a be a constant, $a \neq 0.$

a) By using Factor Theorem, explain why $x-a$ is a factor of $x^3-a^3.$

b) State the other quadratic factor of x^3-a^3 and show that this quadratic factor cannot be factorised further.

Question 32:

Let $\mathrm{f}(x)=2x^3+ax^2+bx-2,$ where *a* and *b* are constants. It is given that when $\mathrm{f}(x)$ is divided by $x^2+x-2,$ the quotient is $\mathrm{Q}(x)$ and the remainder is $x-2.$

a) Express $\mathrm{f}(x)$ in terms of $\mathrm{Q}(x),$ x^2+x-2 and $x-2.$

b) Find the values of *a* and *b*.

c) Find $\mathrm{Q}(x).$ Hence, find the remainder when $\mathrm{f}(x)$ is divided by $\mathrm{Q}(x).$

Question 33:

Express $\dfrac{x^3}{(x+2)(x^2+4)}$ in partial fractions.

Question 34:

Given that $3x^3+2x^2+x-4=A(x+1)(x-1)+B(x-1)+Cx(x^2-1)+D$ for all values of *x*, find the values of *A*, *B*, *C* and *D*.

Question 35:

Express $\dfrac{7x+8}{(2+3x)^2(1-x)}$ in partial fractions.

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Question 36:

The polynomial $3x^3 + ax^2 - 5x + b$, where *a* and *b* are constants, has a remainder *R* when divided by $x - 1$ and has a remainder of 7*R* when divided by $x + 2$. The polynomial $3x^3 + ax^2 - 5x + b$ is exactly divisible by $x - 2$.

a) Show that $a = -4$ and $b = 2$.

b) Solve $3x^3 + ax^2 - 5x + b = 0$.

c) **Hence**, solve $\frac{3}{8}x^3 - x^2 - \frac{5}{2}x + 2 = 0$.

Question 37:

a) Factorise completely $64x^3 - (x-1)^3$.

b) Express $\frac{4x^3 - 7x^2 + 2x - 6}{(x-2)(2x^2+5)}$ in partial fractions.

Question 38:

The function g is defined as $g(x) = 6x^3 + px^2 - 10x + q$, where *p* and *q* are constants. Given that $g(x)$ is exactly divisible by $(2x-1)$ and it leaves a remainder of -27 when divided by $(x+1)$.

a) Show that $p = -47$ and $q = 16$.

b) Hence, solve $g(x) = 0$.

Question 39:

Express $\frac{5x^2 - 4x + 5}{(x-1)(x^2+2)}$ in partial fractions.

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Question 40:

The polynomial $P(x) = 2x^3 - 21x^2 + hx + 60$ leaves a remainder of 45 when divided by $x - 1$.

a) Show that $h = 4$.

b) Factorise P(x) completely.

c) Hence find the solution of $P(2x) = 0$.

Question 41:

a) When $x^2 - bx + 3$ and $2b - x$ are each divided by $x - a$, the remainders are 30 and -3 respectively. Find the values of a and of b.

b) Express $\frac{2x^2+3x+41}{(x+1)(x^2+9)}$ in partial fractions.

Question 42:

Show that $(x - 4)$ is a factor of $x^3 - 3x^2 - x - 12$.

Hence, factorise $x^3 - 3x^2 - x - 12$ completely and show that $(x - 4)$ is the only linear factor.

Question 43:

The function $f(x) = x^3 - 6x^2 + px + q$, where p and q are constants, is exactly divisible by $x - 3$ and leaves a remainder of -55 when divided by $x + 2$.

a) Find the values of p and q.

b) Solve the equation f(x) = 0.

Question 44:

Given that $Ax^4 + 3x^3 - 18x + C = (x^2 + 1)(2x^2 + Bx - 2) - 3(7x - 5)$ for all values of x, find the values of A, B and C.

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Question 45:

Express the following in partial fractions

a) $\dfrac{4x+1}{(x-1)(x+1)(x+2)}$

b) $\dfrac{3x-2}{(x^3+x)}$

Question 46:

It is given that $\mathrm{f}(x)=2x^3+3x^2-3x-2$.

a) Factorise $\mathrm{f}(x)$ completely.

b) Hence, solve the equation $2(2a+5)^3+3(2a+5)^2=6a+17$.

Question 47:

Express $\dfrac{-x^3+4x^2+2x+16}{(x^2+2)(1-x)}$ in partial fractions.

Explain why $x=1$ is not a valid value for the expression.

Question 48:

a) Given that $6x^3+2x^2-4x-6=(x^2+x-2)(6x-2A)+2B(x-1)+2C$ for all real values of *x*, find the values of *A*, *B* and *C*.

b) The remainder when the expression mx^3+2x^2-nx+7 is divided by $x+2$ is twice the remainder when the same expression is divided by $x-1$.

i) Show that $4n-10m=3$.

ii) Given further that $mn=1$ and $m<n$, find the value of *m* and of *n*.

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Question 49:

Express $\frac{4x^3+8x^2+6x+1}{4x^2+4x+1}$ in partial fractions.

Question 50:

The cubic polynomial f(x) leaves a remainder of 48 when divided by $x-1$ and a remainder of $5-x$ when divided by $2x^2-6x-7$. The term containing the highest power of x in f(x) is $2x^3$.

a) Show that f(x) $=2x^3-16x^2+22x+40$.

b) Solve f(x) $=0$.

Question 51:

Express $\frac{x+3}{x^2\left(2x^2+1\right)}$ in partial fractions.

Question 52:

Express $\frac{8x-27}{(x+1)(x^2+6)}$ in partial fractions.

Question 53:

The expressions x^3-2x^2-px+6 and $x^3+x^2+(8-p)x+10$ leave the same remainder when they are divided by $(x+a)$, where a is an integer.

a) Show that $3a^2-8a+4=0$.

b) Find the value of a.

c) Calculate the value of p if the remainder when divided by $(x+a)$ is -20.

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Question 54:

The remainder when the expression $hx^3 + kx^2 + 4x + 4$ is divided by $(x+2)$ is -24.

a) Given that $(x-1)$ is a factor of the expression, show that $h = -1$ and $k = -7$.

b) Hence, solve $-x^3 - 7x^2 + 4x + 4 = 0$, leaving your answers in the form $a \pm b\sqrt{3}$ where applicable.

Question 55:

It is known that the function $f(x) = 2x^3 + px^2 + qx + 2$ is divisible by $(x-1)$ and $(x+2)$.

a) Find the value of p and of q.

b) Hence, find the remainder when $f(x)$ is divided by $2x+1$.

Question 56:

Express $\dfrac{50}{x^3 + 5x^2}$ as the sum of three partial fractions.

Question 57:

Given that $\mathrm{P}(x) = x^6 - 1$ and $\mathrm{Q}(x) = x^4 + ax^3 + bx^2 + x - 18$,

a) show that $(x+1)$ and $(x-1)$ are factors of $\mathrm{P}(x)$.

b) When $\mathrm{Q}(x)$ is divided by $x^2 - 1$, a remainder of $2x - 19$ is obtained. Show that the value of a and b are -2 and 1 respectively.

c) Find the remainder when $3\mathrm{P}(x) + \mathrm{Q}(x)$ is divided by $x^2 - 1$.

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Question 58:

Express the following in partial fraction.

a) $\dfrac{2x+5}{x^2-25}$

b) $\dfrac{7x-12}{(x+2)(3x^2+1)}$