EQUITY
LEARNING PLACE

Additional Math Topical (Polynomial)

Question 1:

Given that $x^5 - 2x^4 + ax^3 + bx^2 - 17x - 2 = (x^2 - 4)Q(x) - x + 2$, for all values of *x*, where $Q(x)$ is a polynomial and *a* and *b* are constants.

a) State the degree of $Q(x)$.

b) Find the value of *a* and of *b*.

Question 2:

Find the remainder when $4x^3 + 4x^2 - 7x + 2$ is divided by $2x + 1$.

i) Factorise completely the cubic polynomial $4x^3 + 4x^2 - 7x + 2$.

ii) Express $\dfrac{8x^2 - 2x + 9}{4x^3 + 4x^2 - 7x + 2}$ in partial fractions.

Question 3:

A cubic polynomial f(*x*) is such that the roots of $\mathrm{f}(x)$ are $\dfrac{1}{2}, 4$ and -3 and it gives a remainder of 60 when it is divided by (*x* + 1). The coefficient of the highest power of *x* is 2*k*.

a) Show that the value of *k* is 2.

b) Express f(*x*) as a cubic polynomial in *x* with integer coefficients.

c) Hence, solve $\mathrm{f}\left(2^y\right) = 0$.

Question 4:

a) Factorise $x^3 - 2x^2 - 4x + 8$ completely.

b) Hence, express $\dfrac{3x^2 - x + 2}{x^3 - 2x^2 - 4x + 8}$ as partial fractions.

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Question 5:

The function f is defined by $\mathrm{f}(x) = 4x^3 - 16x^2 - x + 4$

a) Solve the equation $\mathrm{f}(x) = 0$.

b) Hence, solve $8y^3 - y^2 - 8y + 1 = 0$.

Question 6:

Express in partial fractions

$$\frac{4x^2 - x + 2}{(2x-1)(x-3)}$$

Question 7:

Express $\dfrac{2x^2+5}{(x-2)(x^2+9)}$ as partial fractions.

Question 8:

a) Given that $2x^4 + ax^3 - x^2 + bx - 3$ is divisible by $2x^2 - x - 3$, show that the values of the constant *a* and of *b* are both equal to -1.

b) Hence, deduce the remainder obtained, in the form of $px+q$, when $2x^4 + ax^3 - x^2 + bx - 3$ is divided by $x^2 - 4$.

Question 9:

Express $\dfrac{(x-1)^2}{(x-3)(x^2+2)}$ in partial fractions.

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Question 10:

The expression $ax^3 - 2x^2 + bx + 1$ is exactly divisible by $x - 1$ but leaves a remainder of 12 when divided by $x + 2$.

a) Find the values of a and b.

b) Find the remainder when this expression is divided by $2x - 1$.

Question 11:

a) Show that $(x + 2)$ is not a factor of $6x^3 - x^2 + 2x + 3$.

b) Express $\frac{6x^3 - x^2 + 2x + 3}{x^2 - 1}$ in the form $Ax + B + \frac{C}{x-1} + \frac{D}{x+1}$

Question 12:

Express $\frac{2x - 24}{3x^2 + 13x - 10}$ in partial fractions.

Question 13:

Given that $x^2 + x - 6$ is a factor of $2x^4 + x^3 - ax^2 + bx + a + b - 1$, find the value of a and of b.

Question 14:

Express $\frac{3 - x}{(1 - x^2)(1 + x^2)}$ in partial fractions.

Question 15:

The expression $f(x) = x^3 + ax^2 + bx - 16$ has a factor of $x - 2$, and leaves a remainder of –9 when divided by $x + 1$.

a) Find the value of a and of b.

b) Hence factorise $f(x)$ completely,

c) Solve the equation $f(x) = e^{3x} + ae^{2x} + be^x - 16$.

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Question 16:

Express $\frac{3x-9}{x(x+3)^2}$ in partial fractions.

Question 17:

The term containing the highest power of x in a polynomial f(x) is $2x^3$ and one of the roots of the equation $f(x)=0$ is 1. Given that $2x^2+5x+4$ is a quadratic factor of f(x), find

a) an expression for f(x),

b) the number of real roots of the equation f(x) = 0, justifying your answer.

c) the remainder when f(x) is divided by $x + 3$.

Question 18:

Given that $x^4+2x^3+x^2-7=(x+3)(x-1)Q(x)+ax+b$ for all values of x and that $Q(x)$ is a polynomial,

a) state the degree of $Q(x)$,

b) find the values of a and b and hence state the remainder when $x^4+2x^3+x^2-7$ is divided by x^2+2x-3.

Question 19:

Express $\frac{5x^3+x^2+4x-1}{x^2(x^2+1)}$ in partial fractions.

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Question 20:

The polynomial $f(x) = 2x^3 + 3x^2 + qx - 6$ has a factor $(x - 2)$.

a) Show that $q = -11$.

b) Solve f(*x*) = 0.

c) Hence, solve the equation $16y^3 + 12y^2 - 22y = 6$.

Question 21:

Express $\frac{x^3}{x^2 - 1}$ in partial fractions.

Question 22:

a) The remainders when $ax^3 + bx^2 - 6x + 4$ is divided by $x + 1$ and $x - 2$ are equal. Given that the polynomial is also divisible by $x + 2$, show that $a = 2$ and $b = 0$.

b) Factorise completely $2x^3 - 6x + 4$. Hence, express $\frac{9}{2x^3 - 6x + 4}$ as partial fractions.

Question 23:

Express $\frac{x + 7}{(x - 3)(x + 2)}$ in partial fractions.

Question 24:

Given that f $(x) = 2x^3 + x^2 - 7x - 6$,

a) find the remainder when f (x) is divided by $x + 5$,

b) show that $x + 1$ is a factor of f (x) and hence solve the equation f $(x) = 0$.

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Question 25:

If $2x+1$ is a factor of $f(x)=2x^3+x^2+px-1$, evaluate p and find the remainder when $f(x)$ is divided by $x-2$.

Question 26:

Express $\dfrac{x-5}{(2x-3)(x-2)^2}$ as partial fractions.

Question 27:

a) For what condition of n is $g(x)=x^n-1$ exactly divisible by $x+1$?

b) The term containing the highest power of x in the polynomial $f(x)$ is $2x^3$. Given that x^2-2x+5 is a quadratic factor of $f(x)$ and one of the roots of the equation $f(x)=0$ is -1, find

i) an expression for $f(x)$ in descending powers of x,

ii) the number of real roots of the equation $f(x)=0$, justifying your answer with suitable working.

Question 28:

a) The remainder is 14 when $f(x)=x^3-5x^2-ax+12$ is divided by $x+1$. Show that the value of a is 8.

Hence, factorise $f(x)=x^3-5x^2-ax+12$ if $x-1$ is factor.

b) Express $\dfrac{16x^2+11x+14}{(x-2)(2x+1)^2}$ in partial fractions.

Question 29:

Express $\dfrac{3x^2-1}{x(x-1)^2}$ in partial fractions.

Question 30:

Without the use of a calculator, solve the equation $y^3 + 4y^2 - 8 = 0.$

By using your answer in previous part, or otherwise, solve the equation $8^x + 4^{x+1} = 8.$

Question 37: