AM Topical Question

Surds Solution

1) The area of a triangle is $\left(1+\frac{5\sqrt{5}}{2}\right)$ cm^2. If the length of the base of the triangle is $\left(3+2\sqrt{5}\right)$cm, find, without using a calculator, the height of the triangle in the form of $\left(a+b\sqrt{5}\right)$cm, where *a* and *b* are integers.

$$\left(1+\frac{5\sqrt{5}}{2}\right) = \frac{1}{2}(3+2\sqrt{5})(h)$$

$$2+5\sqrt{5} = (3+2\sqrt{5})h$$

$$h = \frac{2+5\sqrt{5}}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$$

$$h = \frac{6+15\sqrt{5}-4\sqrt{5}-50}{9-20}$$

$$= \frac{11\sqrt{5}-44}{-11}$$

$$h = (4-\sqrt{5})\text{ cm}$$

2) The length of each side of an equilateral triangle is $\dfrac{22}{5-\sqrt{3}}$ cm.

i) Show that the height of the triangle is $\left(\dfrac{3+5\sqrt{3}}{2}\right)$ cm.

ii) Find the area of the triangle in the form $\dfrac{p+q\sqrt{3}}{2}$, where p and q are integers.

2(i)

$$\sin 60 = \frac{h}{\frac{22}{5-\sqrt{3}}}$$

$$\frac{\sqrt{3}}{2} \times \frac{22}{5-\sqrt{3}} = h$$

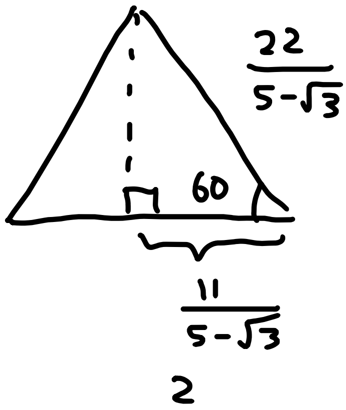
$$h = \frac{11\sqrt{3}}{5-\sqrt{3}} \times \frac{5+\sqrt{3}}{5+\sqrt{3}}$$

$$h = \frac{55\sqrt{3}+33}{25-3}$$

$$= \frac{55\sqrt{3}+33}{22}$$

$$= \frac{3}{2} + \frac{5}{2}\sqrt{3}$$

$$= \frac{3+5\sqrt{3}}{2}$$



2(ii)

$$\text{Area} = \frac{1}{\cancel{2}} \times \frac{\cancel{22}\ 11}{5-\sqrt{3}} \times \frac{3+5\sqrt{3}}{2}$$

$$= \frac{33+55\sqrt{3}}{2(5-\sqrt{3})} \times \frac{5+\sqrt{3}}{5+\sqrt{3}}$$

$$= \frac{165+275\sqrt{3}+33\sqrt{3}+165}{2(25-3)}$$

$$= \frac{330+308\sqrt{3}}{44}$$

$$= \frac{15+18\sqrt{3}}{2}$$

3) The area of a trapezium is $\left(27+\sqrt{5}\right)$ cm². Given that the length of the two parallel sides are $\left(1+\sqrt{5}\right)$ cm and $\left(3\sqrt{5}-5\right)$ cm, express the height of the trapezium in the form $\left(\frac{a+b\sqrt{5}}{c}\right)$ cm, where *a, b* and *c* are integers.

$$(27+\sqrt{5}) = \frac{1}{2}\left[(1+\sqrt{5})+(3\sqrt{5}-5)\right] \times h$$

$$27+\sqrt{5} = \frac{1}{2}(4\sqrt{5}-4) \times h$$

$$27+\sqrt{5} = (2\sqrt{5}-2)h$$

$$h = \frac{27+\sqrt{5}}{2\sqrt{5}-2} \times \frac{2\sqrt{5}+2}{2\sqrt{5}+2}$$

$$= \frac{54\sqrt{5}+10+54+2\sqrt{5}}{16}$$

$$= \frac{56\sqrt{5}+64}{16}$$

$$= \frac{7\sqrt{5}+8}{2}$$

4) Find the value of k such that $\left(\frac{2}{\sqrt{6}}-\frac{\sqrt{150}}{6}+\frac{64}{\sqrt{384}}\right)\times\frac{\sqrt{3}}{5}=k\sqrt{2}$.

$$\left(\frac{2}{\sqrt{6}}-\frac{\sqrt{150}}{6}+\frac{64}{\sqrt{384}}\right)\times\frac{\sqrt{3}}{5}$$

$$=\left(\frac{2}{\sqrt{6}}-\frac{5\sqrt{6}}{6}+\frac{64}{8\sqrt{6}}\right)\times\frac{\sqrt{3}}{5}$$

$$=\frac{2}{5\sqrt{2}}-\frac{\sqrt{18}}{6}+\frac{8}{5\sqrt{2}}$$

$$=\frac{2}{\sqrt{2}}-\frac{\sqrt{2}}{2}$$

$$=\frac{4-2}{2\sqrt{2}}$$

$$=\frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\frac{\sqrt{2}}{2}$$

$$=\frac{1}{2}\sqrt{2}$$

$$\therefore k=\frac{1}{2}$$

5) Express the following in the form $a + b\sqrt{c}$

a) $\left(3 - 5\sqrt{2}\right)^2$,

b) $\dfrac{\sqrt{3}}{\sqrt{3} - \sqrt{2}}$

5a) $(3-5\sqrt{2})^2$

$= 9 - 30\sqrt{2} + 50$

$= 59 - 30\sqrt{2}$

5b) $\dfrac{\sqrt{3}}{\sqrt{3}-\sqrt{2}} \times \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$= \dfrac{3+\sqrt{6}}{1}$

$= 3 + \sqrt{6}$

6) Given that $q = 1 + \sqrt{2}$, evaluate $\frac{q^2 + 3}{q + 2}$

$$\frac{(1+\sqrt{2})^2 + 3}{(1+\sqrt{2}) + 2}$$

$$= \frac{1 + 2\sqrt{2} + 2 + 3}{3 + \sqrt{2}}$$

$$= \frac{6 + 2\sqrt{2}}{3 + \sqrt{2}} \times \frac{3 - \sqrt{2}}{3 - \sqrt{2}}$$

$$= \frac{18 + 6\sqrt{2} - 6\sqrt{2} - 4}{7}$$

$$= \frac{14}{7}$$

$$= 2$$

7) Given that $\frac{a+b\sqrt{5}}{4+3\sqrt{5}} = \frac{4+3\sqrt{5}}{2+\sqrt{5}}$, find the values of a and of b.

$$\frac{a+b\sqrt{5}}{4+3\sqrt{5}} = \frac{4+3\sqrt{5}}{2+\sqrt{5}}$$

$$a+b\sqrt{5} = \frac{(4+3\sqrt{5})^2}{2+\sqrt{5}}$$

$$= \frac{16+24\sqrt{5}+45}{2+\sqrt{5}}$$

$$= \frac{61+24\sqrt{5}}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$$

$$= \frac{122+48\sqrt{5}-61\sqrt{5}-120}{-1}$$

$$= \frac{2-13\sqrt{5}}{-1}$$

$$= 13\sqrt{5}-2$$

$\therefore a=-2$, $b=13$

8) Given that $r = 3 - \sqrt{2}$, express $\frac{5 - r^2}{r + 1}$ in the form $a + b\sqrt{2}$, where a and b are constants.

$$\frac{5 - (3 - \sqrt{2})^2}{(3 - \sqrt{2}) + 1}$$

$$= \frac{5 - (9 - 6\sqrt{2} + 2)}{4 - \sqrt{2}}$$

$$= \frac{6\sqrt{2} - 6}{4 - \sqrt{2}} \times \frac{4 + \sqrt{2}}{4 + \sqrt{2}}$$

$$= \frac{24\sqrt{2} - 24 + 12 - 6\sqrt{2}}{16 - 2}$$

$$= \frac{18\sqrt{2} - 12}{14}$$

$$= -\frac{6}{7} + \frac{9}{7}\sqrt{2}$$

9) The base of a triangle is $\left(3+2\sqrt{7}\right)$ cm and its area is $\left(32+3\sqrt{7}\right)$ cm^2. Find the height of the triangle in the form $\left(a+b\sqrt{7}\right)$.

$$32+3\sqrt{7} = \frac{1}{2} \times (3+2\sqrt{7})(h)$$

$$h = \frac{64+6\sqrt{7}}{3+2\sqrt{7}} \times \frac{3-2\sqrt{7}}{3-2\sqrt{7}}$$

$$= \frac{192+18\sqrt{7}-128\sqrt{7}-84}{-19}$$

$$= \frac{-110\sqrt{7}+108}{-19}$$

$$= \frac{110}{19}\sqrt{7} - \frac{108}{19}$$

10) Without using a calculator, find the value of m such that

$$\left(\frac{3}{\sqrt{15}}+\frac{\sqrt{80}}{5}-\frac{15}{\sqrt{375}}\right)\times\frac{\sqrt{5}}{\sqrt{3}}=m\sqrt{3}.$$

$$\left(\frac{3}{\sqrt{15}}+\frac{\sqrt{80}}{5}-\frac{15}{\sqrt{375}}\right)\times\frac{\sqrt{5}}{\sqrt{3}}$$

$$=\left(\frac{3}{\sqrt{15}}+\frac{4\sqrt{5}}{5}-\frac{15}{5\sqrt{15}}\right)\times\frac{\sqrt{5}}{\sqrt{3}}$$

$$=1+\frac{20}{5\sqrt{3}}-1$$

$$=\frac{4}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$=\frac{4}{3}\sqrt{3}$$

$$\therefore m=\frac{4}{3}$$

11) Prove that $\dfrac{1}{\sqrt{2r-1}+\sqrt{2r+1}} = \dfrac{1}{2}\left(\sqrt{2r+1}-\sqrt{2r-1}\right)$.

Hence, find the exact value of

$$\frac{1}{\sqrt{1}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{9}}+\frac{1}{\sqrt{9}+\sqrt{11}}.$$

$$\frac{1}{\sqrt{2r-1}+\sqrt{2r+1}} \times \frac{\sqrt{2r-1}-\sqrt{2r+1}}{\sqrt{2r-1}-\sqrt{2r+1}}$$

$$= \frac{\sqrt{2r-1}-\sqrt{2r+1}}{(2r-1)-(2r+1)}$$

$$= \frac{\sqrt{2r-1}-\sqrt{2r+1}}{-2}$$

$$= \frac{1}{2}\left(\sqrt{2r+1}-\sqrt{2r-1}\right)$$

$$\frac{1}{\sqrt{1}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + \dots - \frac{1}{\sqrt{9}+\sqrt{11}}$$

$$= \frac{1}{2}(\sqrt{3}-\sqrt{1}) + \frac{1}{2}(\sqrt{5}-\sqrt{3}) + \frac{1}{2}(\sqrt{7}-\sqrt{5}) + \frac{1}{2}(\sqrt{9}-\sqrt{7}) + \frac{1}{2}(\sqrt{11}-\sqrt{9})$$

$$= \frac{1}{2}(-\sqrt{1}) + \frac{1}{2}\sqrt{11}$$

$$= \frac{1}{2}(\sqrt{11}-1)$$

12) Without using a calculator, find the fractions a and b, for which $\dfrac{\sqrt{2}+\sqrt{7}}{\sqrt{21}-\sqrt{6}}$ can be expressed as

$a\sqrt{42}+b\sqrt{3}$.

$$\frac{\sqrt{2}+\sqrt{7}}{\sqrt{21}-\sqrt{6}} \times \frac{\sqrt{21}+\sqrt{6}}{\sqrt{21}+\sqrt{6}}$$

$$= \frac{\sqrt{42}+7\sqrt{3}+2\sqrt{3}+\sqrt{42}}{21-6}$$

$$= \frac{2\sqrt{42}}{15} + \frac{3\sqrt{3}}{5}$$

13) Express $\frac{2}{3-\sqrt{7}} - \left(\sqrt{7}-2\right)^2$ in the form of $a + b\sqrt{7}$ where a and b are integers.

$$\frac{2}{3-\sqrt{7}} - (\sqrt{7}-2)^2$$

$$= \frac{2}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}} - (7-4\sqrt{7}+4)$$

$$= \frac{2(3+\sqrt{7})}{2} - 11 + 4\sqrt{7}$$

$$= -8+5\sqrt{7}$$

14) A cuboid with a square base of length $\left(3-\sqrt{3}\right)$ cm, has a volume of $\left(18\sqrt{3}-24\right)$ cm^3. Find the height of the cuboid in the form $a+b\sqrt{3}$ where *a* and *b* are integers.

$$(18\sqrt{3}-24) = (3-\sqrt{3})^2 h$$

$$(18\sqrt{3}-24) = (9-6\sqrt{3}+3)h$$

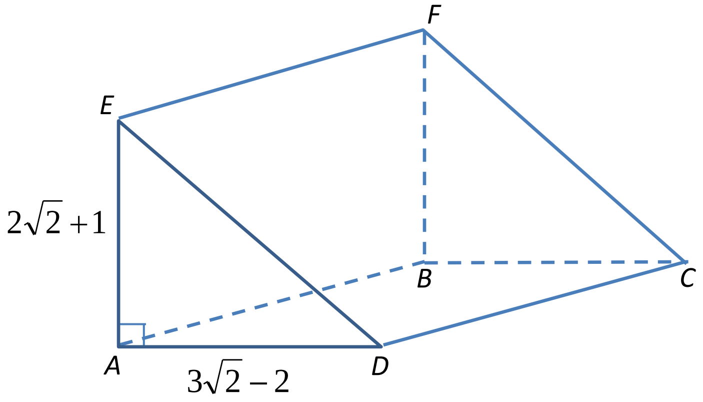
$$\frac{18\sqrt{3}-24}{12-6\sqrt{3}} = h$$

$$h = \frac{3\sqrt{3}-4}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$= \frac{6\sqrt{3}-8+9-4\sqrt{3}}{1}$$

$$= 2\sqrt{3}+1$$

15) The volume of a prism *ABCDEF* is $9+4\sqrt{2}$ cm. The cross section is a right angled triangle where *AE* = $2\sqrt{2}+1$ cm and *AD* = $3\sqrt{2}-2$ cm.



Find

a) the area of triangle *ADE*, leaving your answer in surd form.

b) the length of *DC*, leaving your answer in the form $a+b\sqrt{2}$, where *a* and *b* are integers.

$$\text{Area} = \frac{1}{2}(2\sqrt{2}+1)(3\sqrt{2}-2)$$

$$= \frac{1}{2}(12-\sqrt{2}-2)$$

$$= 5-\frac{1}{2}\sqrt{2}$$

$$9+4\sqrt{2} = \left(5-\frac{1}{2}\sqrt{2}\right)(DC)$$

$$18+8\sqrt{2} = (10-\sqrt{2})\,DC$$

$$DC = \frac{18+8\sqrt{2}}{10-\sqrt{2}} \times \frac{10+\sqrt{2}}{10+\sqrt{2}}$$

$$= \frac{180+98\sqrt{2}+16}{98}$$

$$= 2+\sqrt{2}$$

16) Express $\frac{\sqrt{8}}{4}-\frac{1}{\left(1+2\sqrt{2}\right)^2}$ in the form $a+b\sqrt{2},$ where *a* and *b* are rational numbers.

$$\frac{\sqrt{8}}{4} - \frac{1}{(1+2\sqrt{2})^2}$$

$$= \frac{2\sqrt{2}}{4} - \frac{1}{1+4\sqrt{2}+8}$$

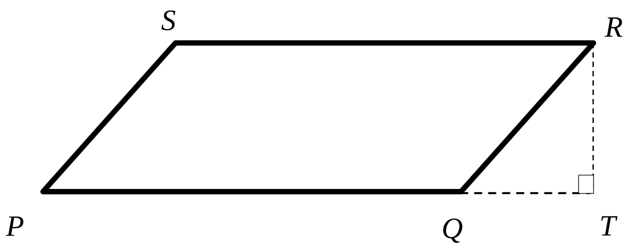
$$= \frac{\sqrt{2}}{4} - \frac{1}{9+4\sqrt{2}} \times \frac{9-4\sqrt{2}}{9-4\sqrt{2}}$$

$$= \frac{\sqrt{2}}{4} - \frac{9-4\sqrt{2}}{81-32}$$

$$= \frac{1}{4}\sqrt{2} - \frac{9}{49} + \frac{4}{49}\sqrt{2}$$

$$= -\frac{9}{49} + \frac{65}{196}\sqrt{2}$$

17) The diagram below shows a parallelogram *PQRS* whose area is $\left(7+8\sqrt{3}\right)\text{cm}^2$.



Given that the length of *QT* is $(5+\sqrt{3})$ cm and *QRT* is an isosceles triangle.

Find, in surd form,

(a) QR^2 in the form $a+b\sqrt{3}$,

(b) *SR* in the form $\dfrac{c+d\sqrt{3}}{2}$.

$$QR^2 = (5+\sqrt{3})^2 + (5+\sqrt{3})^2$$
$$= (25+10\sqrt{3}+3) \times 2$$
$$= 56+20\sqrt{3}$$

$$7+8\sqrt{3} = (SR) \times (5+\sqrt{3})$$
$$SR = \frac{7+8\sqrt{3}}{5+\sqrt{3}} \times \frac{5-\sqrt{3}}{5-\sqrt{3}}$$
$$= \frac{35+40\sqrt{3}-7\sqrt{3}-24}{22}$$
$$= \frac{11+33\sqrt{3}}{22}$$
$$= \frac{1+3\sqrt{3}}{2}$$

18) The area of triangle XYZ is $16+\frac{23}{4}\sqrt{3}$ cm^2 and the length of the side YZ is $5+2\sqrt{3}$ cm. Find the length of the perpendicular from X to YZ in the form of $a+b\sqrt{3}$, where a and b are rational numbers.

$$16+\frac{23}{4}\sqrt{3} = \frac{1}{2}\times(5+2\sqrt{3})\times h$$

$$64+23\sqrt{3} = (10+4\sqrt{3})h$$

$$h = \frac{64+23\sqrt{3}}{10+4\sqrt{3}} \times \frac{10-4\sqrt{3}}{10-4\sqrt{3}}$$

$$= \frac{640+230\sqrt{3}-256\sqrt{3}-276}{100-48}$$

$$= \frac{364-26\sqrt{3}}{52}$$

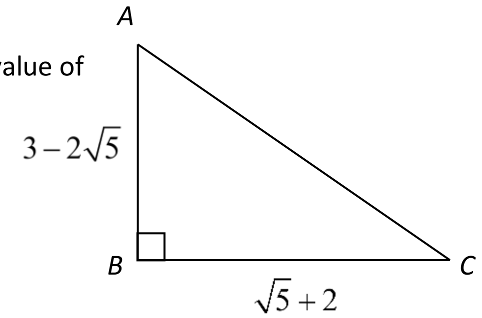
$$= 7-\frac{1}{2}\sqrt{3}$$

19) Triangle *ABC* is a right-angled triangle where $\angle ABC = 90°$.

Without using a calculator, find in the form of $a + b\sqrt{5}$, the value of

a) AC^2,

b) $\tan \angle ACB$.



$$AC^2 = (3-2\sqrt{5})^2 + (\sqrt{5}+2)^2$$

$$= 9 - 12\sqrt{5} + 20 + 5 + 4\sqrt{5} + 4$$

$$= 38 - 8\sqrt{5}$$

$$\tan \measuredangle ACB = \frac{3-2\sqrt{5}}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2}$$

$$= \frac{3\sqrt{5} - 10 - 6 + 4\sqrt{5}}{1}$$

$$= 7\sqrt{5} - 16$$

20) It is given that $\sqrt{3}(x-1) = 3x - 4$. Without using a calculator, find x in the form $a + b\sqrt{3}$, where a and b are rational numbers.

$$\sqrt{3}(x-1) = 3x-4$$

$$\sqrt{3}x - \sqrt{3} = 3x-4$$

$$4-\sqrt{3} = 3x-\sqrt{3}x$$

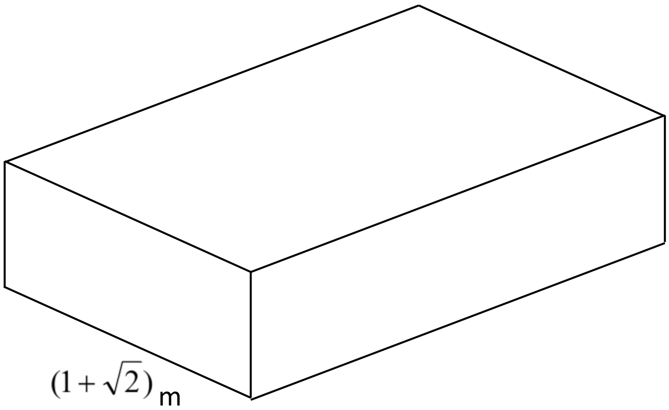
$$\frac{3+\sqrt{3}}{3+\sqrt{3}} \times \frac{4-\sqrt{3}}{3-\sqrt{3}} = x$$

$$x = \frac{12+4\sqrt{3}-3\sqrt{3}-3}{9-3}$$

$$= \frac{9+\sqrt{3}}{6}$$

$$= \frac{3}{2} + \frac{1}{6}\sqrt{3}$$

21) A hollow closed rectangular tank is constructed of thin sheet metal of negligible thickness. The length of the tank is twice the width. The total surface area of the tank is 48 m^2.



If the width is $(1+\sqrt{2})$ m, find the exact value of the height of the tank.

$$48 = (1+\sqrt{2})(2+2\sqrt{2})(2) + 2(1+\sqrt{2})h + (2+2\sqrt{2})h(2)$$

$$24 = (2+4\sqrt{2}+4) + (3+3\sqrt{2})h$$

$$(3+3\sqrt{2})h = 18 - 4\sqrt{2}$$

$$h = \frac{18-4\sqrt{2}}{3+3\sqrt{2}} \times \frac{3-3\sqrt{2}}{3-3\sqrt{2}}$$

$$= \frac{54 - 12\sqrt{2} - 54\sqrt{2} + 24}{9-18}$$

$$= \frac{78 - 66\sqrt{2}}{-9}$$

$$= \frac{22}{3}\sqrt{2} - \frac{26}{3}$$

22a) Without using a calculator, simplify $\frac{3}{\sqrt{3}}\left(\frac{3}{\sqrt{6}}-\frac{\sqrt{384}}{3}+\frac{\sqrt{50}}{2\sqrt{3}}\right)$.

22b) The area of a rectangle is $3\sqrt{6}\ \text{cm}^2$. Its length and breadth are $\frac{a+b\sqrt{3}}{2}$ cm and $\left(\sqrt{6}-\sqrt{2}\right)$cm respectively.

Without using a calculator, find the values of the integers *a* and *b*.

22a) $\frac{3}{\sqrt{3}}\left(\frac{3}{\sqrt{6}}-\frac{\sqrt{384}}{3}+\frac{\sqrt{50}}{2\sqrt{3}}\right)$

$= \frac{3}{\sqrt{3}}\left(\frac{3}{\sqrt{6}}-\frac{8\sqrt{6}}{3}-\frac{5\sqrt{2}}{2\sqrt{3}}\right)$

$= \frac{9}{3\sqrt{2}} - 8\sqrt{2} - \frac{15\sqrt{2}}{6}$

$= \frac{3}{\sqrt{2}} - 8\sqrt{2} - \frac{5\sqrt{2}}{2}$

$= \frac{3}{2}\sqrt{2} - 8\sqrt{2} - \frac{5}{2}\sqrt{2}$

$= -9\sqrt{2}$

22b)

$3\sqrt{6} = (\sqrt{6}-\sqrt{2}) \times l$

$l = \frac{3\sqrt{6}}{\sqrt{6}-\sqrt{2}} \times \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}$

$l = \frac{18+3\sqrt{12}}{6-2}$

$= \frac{18+12\sqrt{3}}{4}$

$= \frac{9+6\sqrt{3}}{2}$

$\therefore a=9,\ b=6$

23) Given that $\sqrt{6}=\left(\sqrt{6}-2\right)x-2$, evaluate $\dfrac{x^2+1}{x}$ without the use of a calculator.

$$\sqrt{6}=(\sqrt{6}-2)x-2$$

$$\frac{\sqrt{6}+2}{\sqrt{6}-2}=x$$

$$x=\frac{\sqrt{6}+2}{\sqrt{6}-2}\times\frac{\sqrt{6}+2}{\sqrt{6}+2}$$

$$=\frac{6+4\sqrt{6}+4}{6-4}$$

$$=5+2\sqrt{6}$$

$$\frac{x^2+1}{x}$$

$$=\frac{(5+2\sqrt{6})^2+1}{(5+2\sqrt{6})}$$

$$=\frac{25+20\sqrt{6}+24+1}{5+2\sqrt{6}}\times\frac{5-2\sqrt{6}}{5-2\sqrt{6}}$$

$$=(50+20\sqrt{6})(5-2\sqrt{6})$$

$$=250+100\sqrt{6}-100\sqrt{6}-240$$

$$=10$$

24) Solve the following equations.

$$\frac{\sqrt{x-4}}{2}=2-\frac{1}{2}\sqrt{2x-1}$$

$$\frac{\sqrt{x-4}}{2} = 2-\frac{1}{2}\sqrt{2x-1}$$

$$\sqrt{x-4} = 4-\sqrt{2x-1}$$

$$x-4 = 16-8\sqrt{2x-1}+2x-1$$

$$8\sqrt{2x-1} = x+19$$

$$64(2x-1) = x^2+38x+361$$

$$128x-64 = x^2+38x+361$$

$$x^2-90x+425=0$$

$$(x-5)(x-85)=0$$

$x=5$ or $x=85$

(N)A

25) A cylinder has a radius of $\left(\sqrt{10}-\sqrt{2}\right)$ cm and a height of *h* cm. The volume of the cylinder is $\left(3+2\sqrt{5}\right)\pi$ cm³. **Without using a calculator**, show that *h* can be expressed as $a+b\sqrt{5}$, where *a* and *b* are rational numbers.

$$(3+2\sqrt{5})\cancel{\pi} = \cancel{\pi}(\sqrt{10}-\sqrt{2})^2 h$$

$$h = \frac{3+2\sqrt{5}}{10-2\sqrt{20}+2}$$

$$= \frac{3+2\sqrt{5}}{12-4\sqrt{5}} \times \frac{12+4\sqrt{5}}{12+4\sqrt{5}}$$

$$= \frac{36+24\sqrt{5}+12\sqrt{5}+40}{144-80}$$

$$= \frac{76+36\sqrt{5}}{64}$$

$$= \frac{19}{16}+\frac{9}{16}\sqrt{5}$$

26) A triangle ABC in which $AB = AC$ has an area of 46 cm^2. Given that the base BC is $\left(8\sqrt{3}-2\sqrt{2}\right)$ cm, find in rationalized surd form,

a) the height of the triangle

b) the perimeter of the triangle

a) $46 = \frac{1}{2}(8\sqrt{3}-2\sqrt{2}) \times h$

$46 = (4\sqrt{3}-\sqrt{2})\, h$

$h = \frac{46}{4\sqrt{3}-\sqrt{2}} \times \frac{4\sqrt{3}+\sqrt{2}}{4\sqrt{3}+\sqrt{2}}$

$= \frac{184\sqrt{3}+46\sqrt{2}}{48-2}$

$= 4\sqrt{3}+\sqrt{2}$

b) $AB^2 = \left(\frac{8\sqrt{3}-2\sqrt{2}}{2}\right)^2 + (4\sqrt{3}+\sqrt{2})^2$

$= (4\sqrt{3}-\sqrt{2})^2 + (4\sqrt{3}+\sqrt{2})^2$

$= 48 - 8\sqrt{6} + 2 + 48 + 8\sqrt{6} + 2$

$= 100$

$AB = 10$

Perimeter $= 10 + 10 + 8\sqrt{3} - 2\sqrt{2}$

$= 20 + 8\sqrt{3} - 2\sqrt{2}$

27) Without using a calculator, find the integers *a* and such that $\dfrac{3\sqrt{5}-1}{\sqrt{5}+2}-\dfrac{15}{\sqrt{5}}=a+b\sqrt{5}.$

$$\frac{3\sqrt{5}-1}{\sqrt{5}+2}-\frac{15}{\sqrt{5}}$$

$$=\frac{(3\sqrt{5}-1)(\sqrt{5}-2)}{1}-\frac{15\sqrt{5}}{5}$$

$$=15-7\sqrt{5}+2-3\sqrt{5}$$

$$=17-10\sqrt{5}$$

$$\therefore a=17,\ b=-10$$

28) Without using a calculator, simplify $(3\sqrt{7}-2)(5+\sqrt{7})$ in the form $a+b\sqrt{7}$, where a and b are integers.

$(3\sqrt{7}-2)(5+\sqrt{7})$

$= 15\sqrt{7}-10+21-2\sqrt{7}$

$= 11+13\sqrt{7}$

29) Show that $\frac{1}{\sqrt{q-1}+\sqrt{q}}$ can be expressed as $-\sqrt{q-1}+\sqrt{q}$.

Hence, find the value of $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+....+\frac{1}{\sqrt{15}+\sqrt{16}}$.

$$\frac{1}{\sqrt{q-1}+\sqrt{q}} \times \frac{\sqrt{q-1}-\sqrt{q}}{\sqrt{q-1}-\sqrt{q}}$$

$$= \frac{\sqrt{q-1}-\sqrt{q}}{q-1-q}$$

$$= -\sqrt{q-1}+\sqrt{q}$$

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \dots + \frac{1}{\sqrt{15}+\sqrt{16}}$$

$$= -\sqrt{1}+\sqrt{2}-\sqrt{2}+\sqrt{3}+\dots-\sqrt{15}+\sqrt{16}$$

$$= -\sqrt{1}+\sqrt{16}$$

$$= -1+4$$

$$= 3$$

30) Without using a calculator, simplify $\frac{\left(\sqrt{5}\right)^3-\left(\sqrt{3}\right)^3}{\sqrt{5}-\sqrt{3}}-\frac{\left(\sqrt{5}\right)^3+\left(\sqrt{3}\right)^3}{\sqrt{5}+\sqrt{3}}$, leaving your answer in exact form.

$$\frac{(\sqrt{5})^3-(\sqrt{3})^3}{\sqrt{5}-\sqrt{3}}-\frac{(\sqrt{5})^3+(\sqrt{3})^3}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{(\sqrt{5}-\sqrt{3})(\sqrt{5}^2+\sqrt{5}\sqrt{3}+\sqrt{3}^2)}{\sqrt{5}-\sqrt{3}}-\frac{(\sqrt{5}+\sqrt{3})(\sqrt{5}^2-\sqrt{5}\sqrt{3}+\sqrt{3}^2)}{\sqrt{5}+\sqrt{3}}$$

$$=5+\sqrt{15}+3-(5-\sqrt{15}+3)$$

$$=2\sqrt{15}$$